Kinematic Synthesis and Analysis of the Rack and Pinion Multi-Purpose Mechanism

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The general procedure for synthesizing the rack and pinion mechanism up to seven precision conditions is developed. To illustrate the method, the mechanism has been synthesized in closed form for three precision conditions of path generation, two positions of function generation, and a velocity condition at one of the precision points. This mechanism has a number of advantages over conventional four bar mechanisms. First, since the rack is always tangent to the pinion, the transmission angle is always 90 deg minus the pressure angle of the rack. Second, with both translation and rotation of the rack occurring, multiple outputs are available. Other advantages include the generation of monotonic functions for a wide variety of motion and nonmonotonic functions for a full range of motion as well as nonlinear amplified motions. In this work the mechanism is made to satisfy a number of practical design requirements such as completely rotatable input crank and others. By including the velocity specification, the designer has considerably more control of the output motion. The method of solution developed in this work uses the complex number method of mechanism synthesis. A numerical example is included.

Introduction

The rack and pinion mechanism has been studied by Meyer (1965), Wilt and Sandor (1969), Kinzel and Chen (1984) as well as Gibson (1984), Hofmeister (1984), Claudio (1986), and Duschl (1987) in collaboration with the second author of this paper. The material presented in this paper extends the work done by Duschl and Kramer (1987) on the solution of six precision conditions (combined path and function generation for three positions) with the addition of prescribed velocity at one of the precision points. As shown in Table 1, the rack and pinion mechanism can be synthesized for a maximum of seven precision conditions with specified values for various combinations of coupler point position, its velocity, rotation of the pinion, and its angular velocity at one or more design positions. The general form of synthesis equations for each one of the four different types of precision conditions is shown in Table 2. The solution procedure depends upon the number of available free choices and is slightly different from one case to another. The present work concentrates on the case (shown prominently in Table 1) in which the mechanism generates a path passing through the three precision points satisfying the specified velocity at one of the three points together with a prescribed finite rotation of the pinion.

The rack and pinion mechanism (Fig. 1) is composed of an input crank, $\mathbf{Z}_2$, whose rotation is specified by the mechanism designer. The rack, $\mathbf{Z}_4$, is in nonslip contact with the pinion such that it rotates by γ_j and translates. The offset, $\mathbf{Z}_3$, is rigidly connected to $\mathbf{Z}_4$ and allows for generality but in some cases may be omitted. The pinion, whose radius vector is $\mathbf{Z}_5$, rotates as the rack rotates and translates. Vector $\mathbf{Z}_6$ defines the tracer point with respect to the tip of the offset and is rigidly attached to the rack. The fixed link, $\mathbf{Z}_1$, connects the two fixed pivots.

This specialized mechanism is similar to the prismatic mechanism (Tsai and Soni, 1979, and Shigley and Uicker, 1980) in that vectors $\mathbf{Z}_3$, $\mathbf{Z}_4$, and $\mathbf{Z}_5$ all rotate with the same angle, since $\mathbf{Z}_4$ is always perpendicular to $\mathbf{Z}_5$. The rack and pinion mechanism, however, produces an additional output, ψ_j, which is

Table 1 Number of precision conditions attainable for 0, 1, and 2 arbitrary choices

PATH Generation	FUNCTION Generation	PATH w Velocity	FUNCTION w Velocity	Arbitrary Choices	Total Precision Conditions
3	3	1	0	0	7
3	2	1	1	0	7
4	2	0	1	0	7
4	0	1	0	0	5
5	0	0	0	0	5
0	5	0	0	0	5
3	2	1	0	1	6
4	2	0	0	1	6
2	2	1	1	2	6
3	3	0	0	2	6
4	0	0	0	2	4
3	2	0	1	2	6

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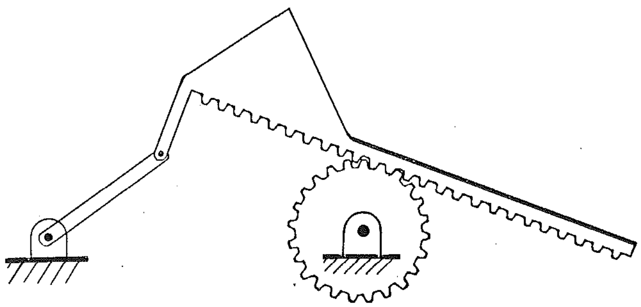

Fig. 1

the rotation of the pinion. The rack and pinion mechanism has industrial applications in the packaging industry as well as toys and other leisure equipment. The inversion of the mechanism where the pinion is the driver, has been used in mechanical aircraft control devices, hospital and laboratory equipment, rack and pinion automotive steering linkages (Zarak and Townsend, 1983) and several tasks on manufacturing assembly lines.

Method of Solution

It is very convenient to use complex number method (Sandor, 1959) to synthesize this mechanism. The vector representation of the rack and pinion mechanism along with coordinate axes are shown in Fig. 2 in the initial design position. The input crank rotations, φ_j, and the position vectors, $\mathbf{R}_j$, are known because of the path specification, where $j=1$, 2, and 3. The rotation of the pinion, ψ_j, and the angular velocity of the input crank, $\dot{\varphi}_j$, are also known. The velocity at the precision point is prescribed as $\dot{\mathbf{R}}_3$. The magnitudes of all the links are unknown but the orientations of $\mathbf{Z}_3$ and $\mathbf{Z}_5$ relative to $\mathbf{Z}_4$ are known. Offset, $\mathbf{Z}_3$, is rigidly connected to $\mathbf{Z}_4$ at a right angle. Physically speaking, the offset need not be perpendicular to $\mathbf{Z}_4$ but kinematically speaking there will always be a perpendicular vector which can be drawn. Also, since $\mathbf{Z}_5$ is perpendicular to $\mathbf{Z}_4$ due to the rack's tangency to the pinion, the following relationships are true (Gibson and Kramer, 1984):

$$\mathbf{Z}_3=\mathbf{Z}_4 h_3 e^{i(\pi/2)}=\mathbf{Z}_4 h_3 i \quad (1)$$

$$\mathbf{Z}_5=\mathbf{Z}_4 h_5 e^{i(-\pi/2)}=\mathbf{Z}_4 h_5 (-i) \quad (2)$$

where the scalar unknown quantities h_3 and h_5 are:

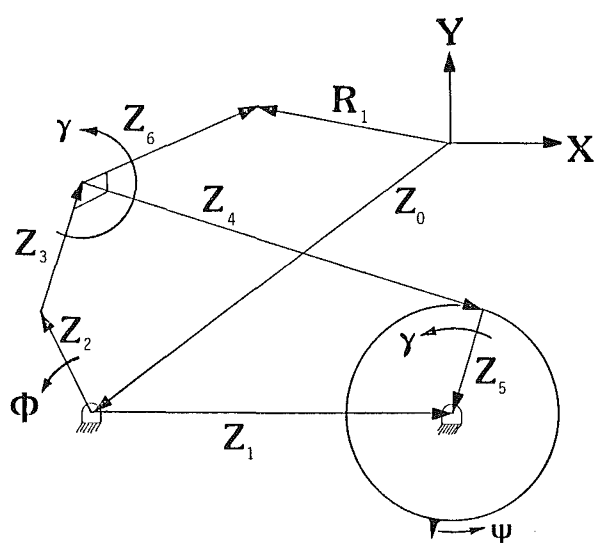

Fig. 2

$$h_3=|\mathbf{Z}_3|/|\mathbf{Z}_4| \ \&$$

$$h_5=|\mathbf{Z}_5|/|\mathbf{Z}_4| \quad (3)$$

Although h_3 and h_5 are ratios of magnitudes, either may be negative to indicate a direction opposite to that assumed in Eqs. (1) and (2). Since $\mathbf{Z}_3$ and $\mathbf{Z}_6$ are rigidly connected, the following definition is adopted for simplicity:

$$\mathbf{Z}_{36}=\mathbf{Z}_3+\mathbf{Z}_6 \quad (4)$$

Since the magnitude of link $\mathbf{Z}_4$ changes from position to position, a stretch ratio is defined as the ratio of the magnitude of $\mathbf{Z}_4$ in its jth position to its initial magnitude so that

$$k_j=\frac{|\mathbf{Z}_{4j}|}{|\mathbf{Z}_4|} \quad (5)$$

From Fig. 2, the position loop closure equation for loop 1 in the initial position can be written as:

$$\mathbf{Z}_0+\mathbf{Z}_2+\mathbf{Z}_{36}=\mathbf{R}_1 \quad (6)$$

Additional loop closure equations can be written for any jth position which represents the general displaced path position. They are:

$$\mathbf{Z}_0+\mathbf{Z}_2 e^{i\varphi_j}+\mathbf{Z}_{36} e^{i\gamma_j}=\mathbf{R}_j \quad \text{for } j=2,\ 3 \quad (7)$$

Subtracting the initial position from the general position yields the following displacement equation:

$$\mathbf{Z}_2(e^{i\varphi_j}-1)+\mathbf{Z}_{36}(e^{i\gamma_j}-1)=(\mathbf{R}_j-\mathbf{R}_1) \quad \text{for } j=2,\ 3. \quad (8)$$

For loop 2, the function loop closure equation can be written for the initial position as:

$$\mathbf{Z}_2+\mathbf{Z}_3+\mathbf{Z}_4+\mathbf{Z}_5=\mathbf{Z}_1 \quad (9)$$

For any jth position,

$$\mathbf{Z}_2 e^{i\varphi_j}+\mathbf{Z}_3 e^{i\gamma_j}+k_j\mathbf{Z}_4 e^{i\gamma_j}+\mathbf{Z}_5 e^{i\gamma_j}=\mathbf{Z}_1 \quad \text{for } j=2,\ 3. \quad (10)$$

Subtracting the initial position from the general position and using Eqs. (1) and (2) yield:

$$\mathbf{Z}_2(e^{i\varphi_j}-1)+\mathbf{Z}_4[(h_3-h_5)i(e^{i\gamma_j}-1) + e^{i\gamma_j}k_j-1]=0 \ \text{for } j=2,\ 3 \quad (11)$$

Referring to Fig. 3, the pinion and the rack rotations can be related using the principle of superposition as noticed by Gibson and Kramer (1984):

$$\psi_3=\gamma_3+\frac{k_3-1}{h_5} \quad (12)$$

where ψ_3 is the prescribed rotation of the rack from first position to the third position. Taking the time derivative of Eqs. (7) and (10) for $j=3$, yields:

$$\mathbf{Z}_2 e^{i\varphi_3} i\dot{\varphi}_3+\mathbf{Z}_{36} e^{i\gamma_3} i\dot{\gamma}_3=\dot{\mathbf{R}}_3 \quad (13)$$

$$\mathbf{Z}_2 e^{i\varphi_3} i\dot{\varphi}_3+\mathbf{Z}_4(k_3+i h_3-i h_5)e^{i\gamma_3} i\dot{\gamma}_3+\mathbf{Z}_4 \dot{k}_3 e^{i\gamma_3}=0 \quad (14)$$

It should be noted that $\dot{\mathbf{R}}_3$ is the specified velocity at the third precision point. In actuality, the velocity condition could have just as easily been specified at the first or second position. The only difference is a change in subscript.

Equation (7), (11), (12), (13), and (14) totally represent 13 scalar equations in 14 scalar unknowns ($\mathbf{Z}_2$, $\mathbf{Z}_{36}$, $\mathbf{Z}_4$, h_3, h_5, k_2, k_3, γ_2, γ_3, $\dot{\gamma}_3$, and $\dot{k}_3$). Hence this system of equations is solvable with one arbitrary choice.

Equations (8) and (13) can be transformed into matrix form as follows:

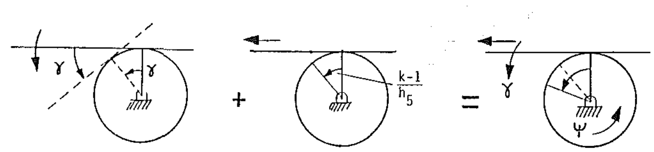

Fig. 3

$$\begin{bmatrix} (e^{i\varphi_2}-1) & (e^{i\gamma_2}-1) & (\mathbf{R}_2-\mathbf{R}_1) \\ (e^{i\varphi_3}-1) & (e^{i\gamma_3}-1) & (\mathbf{R}_3-\mathbf{R}_1) \\ (e^{i\varphi_3}i\dot{\varphi}_3) & (e^{i\gamma_3}i\dot{\gamma}_3) & (\dot{\mathbf{R}}_3) \end{bmatrix} \begin{Bmatrix} \mathbf{Z}_2 \\ \mathbf{Z}_{36} \\ -1 \end{Bmatrix} = 0 \qquad (15)$$

The above system of equations was solved for γ_2, $\dot{\gamma}_3$, $\mathbf{Z}_2$, and $\mathbf{Z}_{36}$ with γ_3 as the arbitrary choice. The above system represents 3 positions and one velocity, and the form of the equations is very similar to the well known Burmester Four-Point problem. Here, the unknowns are γ_2 and $\dot{\gamma}_3$. The unknown γ_2 is eliminated using the complex conjugate property. This leaves one scalar quadratic equation in $\dot{\gamma}_3$. The solution is readily found. Back substitution into Eq. (6) will yield Z_0.

By using Eq. (12) and by rearranging Eqs. (11) and (14) we get:

$$\mathbf{Z}_2(e^{i\varphi_2}-1) + \mathbf{Z}_2[i(e^{i\gamma_2}-1)h_3 - i(e^{i\gamma_2}-1)h_5 + e^{i\gamma_2}k_2 - 1] = 0 \qquad (16)$$

$$\mathbf{Z}_2(e^{i\varphi_3}-1) + \mathbf{Z}_4[i(e^{i\gamma_3}-1)h_3 + ((\psi_3-\gamma_3)e^{i\gamma_3} - i(e^{i\gamma_3}-1))h_5 + (e^{i\gamma_3}-1)] = 0 \qquad (17)$$

$$\mathbf{Z}_2 e^{i\varphi_3} i\dot{\varphi}_3 + \mathbf{Z}_4 e^{i\gamma_3}[h_3 i^2\dot{\gamma}_3 + h_5 i\dot{\gamma}_3(\psi_3-\gamma_3-i) + \dot{k}_3 + i\dot{\gamma}_3] = 0 \qquad (18)$$

Eliminating $\mathbf{Z}_4$ from the above equations and rearranging terms yield:

$$k_2(\mathbf{T}_2 e^{i\gamma_2}) + h_3(\mathbf{T}_2\mathbf{P}_2 + \dot{\gamma}_3) - h_5(\mathbf{P}_2\mathbf{T}_2 + \mathbf{S}) - \dot{k}_3 = (i\dot{\gamma}_3 + \mathbf{T}_2) \qquad (19)$$

$$0 + h_3(\mathbf{T}_3\mathbf{P}_3 + \dot{\gamma}_3) + h_5(\mathbf{T}_3\mathbf{Q} - \mathbf{S}) - \dot{k}_3 = i\dot{\gamma}_3 + \mathbf{T}_3(1-e^{i\gamma_3}) \qquad (20)$$

where

$$\mathbf{P}_j = i(e^{i\gamma_j}-1), \ \mathbf{T}_j = \frac{(e^{i\varphi_3} i\dot{\phi}_3)}{(e^{i\varphi_j}-1)e^{i\gamma_3}} \quad \text{for } j=2, 3$$

$$\mathbf{Q} = (\psi_3-\gamma_3)e^{i\gamma_3} - \mathbf{P}_3$$

and $\quad \mathbf{S} = i\dot{\gamma}_3(\psi_3-\gamma_3-i)$.

By equating real and imaginary parts in the above two independent complex equations, k_2, h_3, h_5, and k_3 can be readily found because the equations are linear in the four unknowns. By substituting the known values into Eq. (12) and (16) k_3 and $\mathbf{Z}_4$ can be obtained. The remainder of the link vectors can be found by back substituting into Eqs. (1), (2), (4) and (10).

Table 2 Governing equations for the mechanism

$$Z_0 + Z_2\, e^{i\phi_j} + Z_{36}\, e^{i\gamma_j} = R_j$$

$$Z_2\, e^{i\phi_j} + Z_4\, e^{i\gamma_j}(h_3\, i + k_j - h_5\, i) = Z_1$$

$$k_j = 1 + h_5(\psi_j - \gamma_j)$$

$$Z_2\, e^{i\phi_j}\, i\dot{\phi}_j + Z_{36}\, e^{i\gamma_j}\, i\dot{\gamma}_j = \dot{R}_j$$

$$\dot{k}_j = h_5(\dot{\psi}_j - \dot{\gamma}_j)$$

Design Restrictions

Since γ_3 is a free choice in the solution procedure, theoretically an infinite number of solutions are possible. However many solutions are rejected because of unreasonable link length ratio, too large an offset for the rack, negative value for at least one of the k's (which implies the sudden flipping of the rack) or if the input crank interferes with the pinion. Claudio and Kramer (1986) noticed that the restrictions to insure that the rack remains in contact with the pinion, and that the input crank does not interfere with the pinion were:

$$|\mathbf{Z}_2| + |\mathbf{Z}_3| < |\mathbf{Z}_1| + |\mathbf{Z}_5| \quad \text{if } h_3\, h_5 > 0 \text{ and,}$$

$$|\mathbf{Z}_2| + |\mathbf{Z}_3| < |\mathbf{Z}_1| - |\mathbf{Z}_5| \quad \text{if } h_3\, h_5 < 0. \qquad (21)$$

The condition for complete rotatability of the crank is given as:

$$|\mathbf{Z}_2| + |\mathbf{Z}_5| < |\mathbf{Z}_1| \qquad (22)$$

If the above requirements are met, then complete rotation of the input crank is insured without the problem of branching.

Analysis

Position analysis of the rack and pinion mechanism was done by Claudio and Kramer (1986). They found that the values of k and γ for any given position of the input crank were:

$$k = \sqrt{(1-|\mathbf{M}|^2)/|\mathbf{N}|^2} \qquad (23)$$

$$\gamma = -\arg(\mathbf{M} + i\mathbf{N}\, k) \qquad (24)$$

where,

$$\mathbf{M} = \frac{\mathbf{Z}_3 + \mathbf{Z}_5}{\mathbf{Z}_1 - \mathbf{Z}_2 e^{i\varphi}} \ \& \ \mathbf{N} = \frac{\mathbf{Z}_4}{\mathbf{Z}_1 - \mathbf{Z}_2 e^{i\varphi}} \qquad (25)$$

For analysis, subscripts are not used since the values of γ, k, ψ, $\dot{k}$, $\dot{\mathbf{R}}$, and $\dot{\gamma}$ correspond to a general input crank rotation, φ. Velocity analysis requires the determination of the first derivatives of the stretch ratio, k, and rack rotation γ, with respect to time for any position of the mechanism. This can be done by substituting the values of k and γ obtained from the position analysis in Eq. (14) and equating the real and imaginary parts. The velocity of the tracer point can then be found directly by using Eq. (13).

Numerical Example

To illustrate the validity of the method developed in this paper, a mechanism has been synthesized for the specifications given below.

- The path vector of the tracer point is described by the following expression:

$$\mathbf{R} = (6 \text{ Cos } \varphi,\ 2 \text{ Sin } \varphi)\text{(inches)}$$

- The pinion rotation is taken to be: $\psi = \varphi + \varphi^2/900$ where φ and ψ are in degrees
- The input crank is assumed to be rotating at a constant angular velocity of 2 rad/sec. Hence, $\dot{\varphi} = 2.0$ in all positions.
- The precision points for path generation are taken at $\varphi = 20$, 60, 170 degs.
- The pinion rotation is specified at the first and third precision points
- The velocity vector at the third precision point is taken as (4, −7.5) inch/sec.

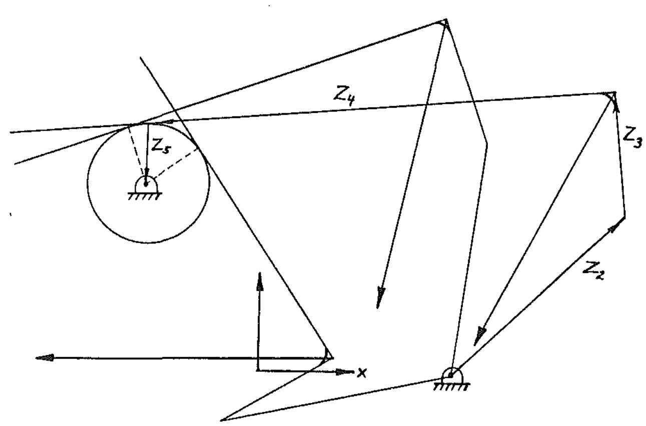

Fig. 4 Rack and pinion mechanism solution

Using the synthesis equations derived above, a computer code was written in Fortran. Since γ_3 is an arbitrary choice, many solutions were obtained. The design restrictions were then applied to reduce the number of solutions. More than half the solutions were rejected because k_2 or k_3 turned out to be negative. Out of the remaining solutions, the one with γ_3 equal to -60 deg was selected. The mechanism solution is shown in Fig. 4 and listed in Table 3. It can be noticed that the directions of the vectors $\mathbf{Z}_3$ and $\mathbf{Z}_5$ are opposite to those assumed because, h_3 and h_5 are negative in this case. The mechanism was then analyzed and the results are tabulated in Table 4. It can be seen that the velocity at the third precision point exactly matches the specified value.

Table 3 Mechanism solution

$\gamma_2 = 15.2°, \gamma_3 = -60.0°$

$\dot{\gamma}_3 = -0.886$

$k_2 = 0.712, k_3 = 0.462$

$h_3 = -0.267, h_5 = -0.127$

$\dot{k}_3 = 1.0173$

$Z_0 = (4.98, 0.08)$

$Z_1 = (-8.0, 4.3)$

$Z_2 = (4.49, 4.10)$

$Z_3 = (-0.40, 3.28)$

$Z_4 = (-12.28, -1.51)$

$Z_5 = (0.19, -1.57)$

$Z_6 = (-3.43, -6.78)$

Conclusion

The rack and pinion mechanism is a versatile mechanism because it can perform path and function generation simultaneously and it has good transmission characteristics since the transmission angle is always equal to 90 deg minus the pressure angle of the rack. In this paper a velocity condition is also taken into consideration which makes this mechanism more useful in practical applications. The synthesis of this single degree of freedom mechanism for multiple output (path and function) makes it very valuable in machine and mechanism design.

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Table 4 Analysis results

Precision Points at: Phi1 = 20.0 deg., Phi2 = 60.0 deg., Phi3 = 170.0 deg.

PHI (Deg)	GAMMA (Deg)	GAMDOT (Rad/s)	K	KDOT	PSI (Deg)	PSIDOT (Rad/s)	POSITION (Inches) Real	POSITION (Inches) Imag	VELOCITY (Inch/s) Real	VELOCITY (Inch/s) Imag
0.00	0.000	0.817	1.104	-0.506	-34.336	4.783	6.315	-0.525	-2.246	7.747
10.00	4.071	0.811	1.056	-0.596	-8.647	5.486	6.045	0.120	-3.904	6.973
20.00	8.104	0.801	1.000	-0.681	20.445	6.143	5.638	0.684	-5.397	5.915
30.00	12.077	0.787	0.937	-0.760	52.696	6.748	5.109	1.145	-6.682	4.608
40.00	15.961	0.765	0.868	-0.832	87.819	7.290	4.479	1.482	-7.727	3.100
50.00	19.713	0.734	0.792	-0.895	125.468	7.756	3.768	1.682	-8.516	1.445
60.00	23.270	0.686	0.712	-0.949	165.226	8.130	3.000	1.732	-9.053	-0.298
70.00	26.535	0.614	0.627	-0.990	206.562	8.381	2.195	1.629	-9.372	-2.060
80.00	29.341	0.498	0.539	-1.015	248.754	8.461	1.368	1.374	-9.566	-3.758
90.00	31.385	0.300	0.450	-1.016	290.719	8.266	0.523	0.978	-9.828	-5.275
100.00	32.070	-0.066	0.363	-0.973	330.593	7.568	-0.361	0.466	-10.588	-6.378
110.00	30.121	-0.802	0.283	-0.844	364.689	5.819	-1.364	-0.105	-12.738	-6.403
120.00	22.825	-2.264	0.221	-0.529	385.144	1.884	-2.658	-0.562	-17.289	-3.256
130.00	6.604	-4.143	0.199	0.042	379.093	-4.475	-4.335	-0.495	-19.710	5.473
140.00	-15.539	-4.314	0.228	0.587	343.855	-8.915	-5.733	0.255	-10.874	9.602
150.00	-33.967	-2.977	0.293	0.869	296.111	-9.796	-6.240	0.859	-1.697	3.387
160.00	-45.537	-1.721	0.375	0.983	247.861	-9.428	-6.189	0.841	2.248	-3.398
170.00	-51.896	-0.886	0.462	1.017	202.111	-8.866	-5.909	0.347	4.000	-7.500
180.00	-54.910	-0.359	0.551	1.013	159.189	-8.305	-5.503	-0.407	5.289	-9.505
190.00	-55.798	-0.020	0.638	0.986	119.052	-7.750	-4.986	-1.273	6.553	-10.179
200.00	-55.293	0.207	0.723	0.942	81.707	-7.185	-4.359	-2.158	7.815	-9.990
210.00	-53.841	0.364	0.803	0.887	47.244	-6.596	-3.624	-3.000	9.003	-9.207
220.00	-51.722	0.477	0.877	0.823	15.801	-5.976	-2.792	-3.753	10.037	-8.000
230.00	-49.121	0.560	0.946	0.750	-12.458	-5.322	-1.879	-4.387	10.851	-6.489
240.00	-46.159	0.622	1.008	0.670	-37.358	-4.633	-0.906	-4.880	11.394	-4.771
250.00	-42.925	0.670	1.063	0.584	-58.735	-3.912	0.101	-5.216	11.632	-2.930
260.00	-39.480	0.707	1.110	0.493	-76.433	-3.162	1.115	-5.390	11.550	-1.044
270.00	-35.869	0.736	1.149	0.398	-90.318	-2.388	2.107	-5.399	11.143	0.815
280.00	-32.129	0.759	1.179	0.300	-100.276	-1.593	3.050	-5.250	10.421	2.579
290.00	-28.286	0.777	1.201	0.199	-106.221	-0.783	3.918	-4.953	9.407	4.189
300.00	-24.363	0.792	1.214	0.096	-108.093	0.035	4.685	-4.525	8.133	5.591
310.00	-20.376	0.802	1.218	-0.007	-105.863	0.857	5.331	-3.985	6.640	6.737
320.00	-16.342	0.811	1.213	-0.110	-99.532	1.675	5.839	-3.358	4.977	7.594
330.00	-12.275	0.816	1.198	-0.213	-89.132	2.483	6.196	-2.669	3.199	8.133
340.00	-8.187	0.819	1.176	-0.313	-74.728	3.275	6.395	-1.948	1.363	8.339
350.00	-4.091	0.819	1.144	-0.411	-56.419	4.044	6.434	-1.223	-0.471	8.209
360.00	0.000	0.817	1.104	-0.506	-34.336	4.783	6.315	-0.525	-2.246	7.747

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