Geometrically Nonlinear Elastic Analysis of Frames with Application to Vision-Based Force-sensing and Mechanics of Plant Stems

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by

P SIVANAGENDRA

DEPARTMENT OF MECHANICAL ENGINEERING
INDIAN INSTITUTE OF SCIENCE
BANGALORE – 560 012
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To

My parents and Teachers
Abstract

The implementation of geometrically nonlinear analysis of the elastic deformation of frame structures for application in two biology related problems is the focus of this project. The manipulation of single biological cells with force feedback is the first application. Towards this, a vision-based force sensing technique is developed using miniature slender arches and beams. The digital images of such manipulating flexible structures captured using a charge-coupled-device (CCD) camera are used to estimate the force(s) of interaction. This involves an inverse problem in elasticity. The general technique where the location, the direction, and the magnitude of the loading are not known is found to be overly sensitive to errors in the measurement of displacements. Hence, a practically useful and accurate technique is developed by assuming the location of the applied force. The results of the numerical experiments are validated with the experimental data obtained using spring-steel prototypes manufactured using electric discharge machining (EDM). The second application of the nonlinear analysis of frames is an investigation related to the compliance of plant stems. By noting that the stems of cereal plants prefer flexibility over stiffness to prevent their uprooting from the ground, a novel deformation-dependent structural optimization problem is formulated. The solution of this problem shows that compliant design is better suited for plant-stems under transverse wind loads. This conclusion is consistent with the experimental data reported in the literature on the genetically modified wheat crops. An experiment is also carried out in the wind-tunnel to verify the deformation results of the finite element code under deformation-dependent loads encountered in plant stems.
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Chapter 1

Introduction

The principles of mechanics and material models together with modern computational tools help in understanding the mechanical behavior of biological objects, which has applications in the areas of medicine, agriculture production, etc. In this report, we consider the application of geometrically nonlinear elastic analysis of frame structures towards vision-based force-sensing technique that is suitable for micromanipulation, and in investigating the role of compliance in plant stems. This chapter gives the necessary background to these two problems. A brief introduction to the micromanipulation is presented in the next section.

1.1 Micromanipulation

The emerging cross-disciplinary field of robotic bio-manipulation of tissue, isolated cells and their contents brings together the traditional mechanical engineering, microsystems (also known as micro-electro-mechanical systems (MEMS)) technology and cellular biomechanics. The main focus of this interdisciplinary research is to develop the tools for precise control of bio-manipulation activities and study the response of biological matter to the mechanical forces. The microsystems technology enables the fabrication of micron-sized tools, sensors and actuators using which the biological cells could be manipulated. Using the design and control techniques, manipulation tasks can be automated.
Automation helps enhance the precision and repeatability of such tasks. This in turn makes the single-cell studies more viable while making them less tedious for the human users [1, 2, 3]. Additionally, automated and controlled manipulation of micron-sized biological entities enables their mechanical characterization, that has implications in biological studies [4]. It is also important to note that this field of micromanipulation also has application in micro-assembly, minimally invasive surgery, micro and adaptive optics, etc. The next section discusses the importance of force-sensing in micromanipulation.

1.2 Force-sensing

Micromanipulation demands high dexterity, which could be facilitated by controlling the forces involved in the manipulation process. Manipulating an object in general requires the ability to observe, position, and physically transform (with force) it. Currently, sufficiently mature microscopy and micro-positioning technologies are available. But they are not adequate for more sophisticated micromanipulation (manipulation that requires high degree of dexterity) because with these techniques only position is modeled and controlled. When manipulating the micro objects, especially delicate structures or biological material which are usually fragile, position control alone is not sufficient in ensuring successful operation and in preventing damage to the object. Force control is often needed to augment the operation in order to achieve better manipulation results. Moreover, in certain applications (such as individual cell based diagnosis or pharmaceutical tests), obtaining force information is beneficial. This would involve probing or reconstructing the state of the micro objects through the knowledge about the micro scale forces of interaction between the manipulator and the object. For example, the zebra-fish eggs at different developmental stages have different stiffness which was characterized by determining the Young’s modulus [4]. Mechanical- property characterization was done by studying the force required to penetrate the egg-envelope at each developmental stage. This shows that micromanipulation with force control is an emerging area, which is likely to become an important component in microsystems technology as well as in biology.
When external forces are applied to a force sensor, its sensing element will deform. This deformation is either detected by measuring the change in certain properties of the sensing element (e.g., change in resistance or capacitance), or it is directly measured by optics based devices (e.g., atomic force microscope). The applied force is calculated from the established calibration between the deformation and a known force. In micro-manipulation, the magnitude of forces may range from 1 nN to 1 µN. Such small forces pose challenge on the design and construction of sensors that can provide measurements with high resolution and high accuracy. To meet these requirements, semiconductor and micro-fabrication techniques have been applied to build sensitive and stable sensing elements. Currently, the types of widely used micro force sensors are of piezoresistive, capacitive, and optical kinds. But all these sensors are either intrusive or are not suitable for micromanipulation in aqueous environment. Thus there is a necessity for minimally-intrusive force-sensing technique.

The present work is devoted towards developing a minimally intrusive vision-based force feedback system that is suitable for micromanipulation studies. This part of the work extends the earlier work by Wang et al. [8] and Greminger et al. [9] on vision-based force-sensing. The vision-based force-sensing method facilitates force feedback in real-time manipulation of micro-objects such as cells. This involves large displacement of elastically deforming structures. The details of the vision based force sensing are presented in Chapter 4. In the next section, the bio-mechanical factors considered in understanding the plant mechanics are presented.

1.3 Plant mechanics

Wind-related crop damage is a major obstacle to cereal production (mainly in wheat) that costs several billion dollars per year [10]. Mechanical damage to the wheat plants is due to wind-induced stress [11], resonance [12], anchorage rotation and buckling [10]. These bio-mechanical factors play an important role in the growth and endurance of wheat plants. Hence, the plant bio-mechanics is an important area of study. While stress, resonance
Chapter 1. Introduction

and buckling are familiar terms to engineers, “anchorage rotation” is a new term. It refers to one of the main modes of failure wherein the plant is uprooted at the ground anchor. If we model the vertically growing plants stem as a cantilever beam, then the anchorage rotation depends on the reaction moment at the ground anchor. Minimizing this moment helps prevent anchorage rotation. Adequate strength and stiffness are also additional criteria while the mass of the material of the stem is limited. Thus, optimization seems to be inherent in plant stems. From the view point of structural optimization this is an interesting problem because the loading pattern here is deformation-dependent, which is rarely considered in the literature. This part of the work is explained in detail in Chapter 5

1.4 Organization of the report

The rest of the report is organized as follows:

• Chapter 2
   A review of the relevant literature is presented in this chapter.

• Chapter 3
   In this chapter, the implementation details of two-node co-rotational beam finite element are presented and validated with benchmark examples.

• Chapter 4
   This chapter is devoted to develop the vision based force-sensing method that is suitable for real-time micromanipulation. To overcome the error-prone estimation of force with the direct method, we propose two new force-sensing methods when the location of the force is known. These methods are validated with the experiments conducted on centimeter-scale semicircular arch.

• Chapter 5
   Plant stems prefer compliant design to withstand heavy wind loads. We posed the
size optimization problem of a beam subjected to deformation-dependent loading to understand the motivation for the compliance of the plant structure.

1.5 Closure

In this chapter, the importance of the nonlinear elastic analysis in the context of biomechanics is introduced. The application of the geometrically nonlinear elastic analysis of frames towards vision-based force sensing and in understanding mechanics involved in the compliance of the plant stems is considered.
Chapter 2

Literature Review

2.1 Micromanipulation

The motivation for micromanipulation stems from two major applications. The first application is concerned with the microassembly of components into a system. Micro components and devices are usually batch-fabricated, but there are some fabrication process such as LIGA (a German acronym for a technique comprising of X-ray lithography, electrodeposition and molding), which requires microassembly of small parts. Many photonic and optical telecommunication components also need assembly at the micro scale [13].

The second application of the micromanipulation is concerned with the need to manipulate and characterize the naturally occurring micro objects such as biological cells and organelle. Researchers in the areas of medicine, biochemistry, physics and engineering have already begun to hold and move not only the individual cells but also intracellular components [7]. In addition to aspiration, laser tweezers, magnetic beads and atomic force microscopy (AFM) techniques are also used in the micromanipulation. In the next section, the aspiration technique used in micromanipulation is discussed.
2.1.1 Intracytoplasmic injection with aspiration

In this section, we consider the aspiration technique used for intracytoplasmic injection. The intracytoplasmic injection involves mainly two tasks. The first task involves capturing and holding the cell, which is performed with a microtrapper. The second task involves, injecting the fluid or DNA or other biological matter into the cell. The schematic representation of the intracytoplasmic injection is shown in Fig. 2.1. Presently, the heating and pulling [14] a glass tube is used for producing the microtrapper and the injector. In this method, the capillary is locally heated with a laser beam over a small region and then it is pulled out until it breaks and leaves a sharp tip. But this heating and pulling method requires expensive micro-pipette pullers with sophisticated control over the temperature and pulling speed to get reproducible geometries. To overcome these difficulties an inexpensive fabrication process based on chemical etching is considered in the present work. This technique is motivated by the recent work of Puygraniar et al.[15] and [16] in the context of shaping optical fiber tips by chemical etching. These probes find applications in optical near field scanning microscopy. In this project the chemical etching technique proposed by Wong et al. [17] is used to fabricate the microprobes. The details of the chemical etching method are presented in Appendix A. In the next section the application of laser tweezers in micromanipulation is described briefly.

![Figure 2.1: Schematic representation of the intracytoplasmic injection](image-url)
2.1.2 Laser tweezers

In addition to the immense applications in engineering and medicine, laser technology is also used to manipulate the single cell and perform surgical operations on it. Bern [18] used laser tweezer to perform manipulation at cellular and subcellular level. Figure 2.2 illustrates the single cell manipulation with laser tweezers. During the operation, a pair of laser tweezer beams (see Fig. 2.2) holds the single cell firmly at one place. The other beams are used to cut a hole into the cell for passing fluid or other biological matter into it or to delete the faulty gene by concentrating the laser beam on it. Implementing the laser technology in micromanipulation is an expensive task and it also produces the thermal side effects on micro-organelle during manipulation. As an inexpensive alternative to this and others, the vision-based force-sensing technique in conjunction with aspiration can be used.
2.1.3 Atomic force microscopy

In addition to the application in material science and metallurgical studies, atomic force microscope (AFM) is used in micromanipulation. In this process, a sharp probe tip attached to the cantilever beam used in AFM is used in single cell studies. Sung et al. [19] used AFM and nano needle to transfer DNA into the cell. In addition to the micromanipulation, the cantilever beam is used as a force sensor based on the deformation. This method overcomes the difficulties associated with placement of piezo-resistive layer embedded into the cantilever beam to sense the force [3]. Although this technique is minimally invasive, it is expensive and requires specially manufactured nano-needle for intracytoplasmic injection. However, this technique is not suitable for manipulation of an object such as cell. Hence, there is a need for an inexpensive manipulation integrated with a force feedback system. This can be achieved if the vision-based force-sensor is integrated with the aspiration tools.

The force feedback is helpful for successful micromanipulation. A review of force sensing methods used in micromanipulation is presented in the next section.

2.2 Force-sensing

Micron sized objects such as biological cells and tissues easily get damaged during their manipulation. To avoid this damage it is helpful to have force-feedback for a human user or an automated tool. Currently, the following force-sensors are widely used at the micro-scale: piezoresistive sensor, capacitive sensor and optical sensor.

The piezoresistive force sensors belong to the semiconductor strain gauge category. In this sensor, the change in resistance of the piezoresistive element due to the deformation of an elastic member is used to measure the force. These sensors find applications in sensing the gripping force involved in micro devices by attaching the piezoresistive element to the gripping surface [5]. Tan et al. [3] used the piezoresistive force sensor for computer controlled micromanipulation. The measurement range and resolution of the force-sensor is varied by selecting the material properties and geometry of the elastic
In capacitive force-sensors, the change in capacitance between the two plates is used as a measure of the force. The capacitative force sensors were successfully implemented in biological studies by Sun et al. [1] with a micro-probe attached to the sensor. In this case, two orthogonal comb drives are used to study the mechanical properties of mouse zona pellucida during the fertilization of an egg cell. But this device is not suitable for direct use in aqueous media.

When the size of the object to be manipulated is very small, it is not possible to integrate the sensing element with the sensor. In such cases, the sensing element is made as an integral part of the manipulator. Optics based techniques are used to sense the force from the deformation of the sensing element associated with the manipulator. In micromanipulations which use AFM as the manipulator, laser-diode and photo-diode are used for sensing the deformation of the AFM cantilever beam. This technique finds application in delivering DNA with a nano-needle attached to the cantilever beam [19]. When an AFM is used in aqueous medium the reflection and refraction of the transmitted light may reduce the accuracy of the force measurement.

The aforementioned force-sensors are either intrusive or not suitable for use in aqueous media. Hence, there is a need for a minimally intrusive force sensing technique in micromanipulation.

The minimally intrusive vision based force-sensing method ([8] and [9]) can be used as an alternative to the above mentioned force sensing techniques. Towards this, the vision-based force-sensing technique [9] is used to characterize the hardening of the zona pellucida of mouse embryo during fertilization by Sun et al. [4]. In this method, a biomembrane point-load model shown in Fig. 2.3 is employed to analyze the deformation of a mouse embryo. Here, $F$ is the force exerted by the micropipette of radius $c$ on the membrane which creates a dimple of depth $w_d$ and radius $r$. In this model, the biomembrane was modeled as a thin elastic film with the assumption that the cytoplasm exerts hydrostatic pressure on the inner surface. This force-sensing technique had the following limitations.
1. The assumption of linear elastic behavior of bio-membrane is not valid for the range of deformations encountered in micromanipulation.

2. This method is limited to determine the magnitude of the force when its location and direction are known in addition to the material properties of the bio-membrane. But the material properties of these membranes are not known a priori.

In this report, more robust vision based force-sensing algorithms are developed. These algorithms are capable of determining the magnitude and direction of the force without the use of material property data of the bio-membrane considered in manipulation. This improvement is carried out by integrating the force-sensing element (elastic member) as an integral part of the manipulator. The details of these methods are presented in Chapter 4. In the next section, we present the related literature on the co-rotational finite element analysis, which forms the basis for force-sensing algorithms developed in this report.
2.3 Co-rotational finite element analysis

In this project slender beams, arches and frames are used as the structural members. When forces are applied they undergo large displacement in the elastic regime. Proper analysis of these structures requires methods that duly account for the large displacement. Belytschko [22] describes the use of a co-rotational total Lagrangian formulation for nonlinear, transient finite element analysis. The co-rotational formulation attaches a local coordinate frame to each element. This frame rotates with the average rigid body rotation of the element. Belytschko [22] observes that the co-rotational formulation simplifies the computation of nodal forces because the effects of rigid body rotations corresponding to large displacement are treated entirely by the transformation between the element and global coordinate systems. Co-rotational formulations are especially suitable for small-strain, large-rotation analysis as the standard small-strain, small-displacement constitutive relations can be applied with respect to the local coordinate system [20]. This follows from the fact that after the rigid-body motion part is eliminated from the total displacement, the deformation-causing part of the total motion is usually small relative to the local coordinate system. In this report, the co-rotational formulation is extended to account for the deformation-dependent loading.

2.4 Plant mechanics

Failure of the plant stem structure is a major loss to the agricultural cereal production. Researchers are trying to understand the mechanics of the plant stems to investigate the reason for the failure. In this context, mechanically induced stress occurs as a natural consequence of environmental conditions as the aerial parts of the plant are moved usually by wind, rain, irrigation, animals or machinery. The effect of mechanically induced stress due to the wind load was studied by Norman [11]. The competing effects of the buckling strength and anchorage strength was studied by Farquhar et al. [10] with linear finite element model. In this analysis, Farquhar et al. [10] posed an optimization problem for minimizing the mass of wheat stem stalk. The resonance also plays a major role in
plant failure. This aspect of the work was carried by Miller [12]. In the afore mentioned cases, the problem was solved using the linear FEA. But the real behavior of the plants shows nonlinear force-deflection relation. The literature on this topic indicates that plants prefer to keep their stems compliant. In order to understand the rationale for the compliance of the plant stem, we consider the size optimization problem of cantilever beam with deformation-dependent loading by considering the different object functions. The details of the optimization problem is given in Chapter 5

2.5 closure

Prior work in the areas of micromanipulation, vision-based force-sensing and plant mechanics were discussed in this chapter. The motivation for the present work is based on the limitations of the prior literature, which was noted here.
Chapter 3

Implementation of Co-rotational Finite Elements for the Geometrically Nonlinear Analysis of frames

3.1 Introduction

In this chapter a two-node co-rotational beam element for the geometrically nonlinear analysis of Euler-Bernoulli beam is presented and is validated with examples. The co-rotational approach is an alternative way of deriving nonlinear finite elements (Crisfield [20], Belytschko [21]), in which the total motion is decomposed into deformational and rigid-body components by defining a local coordinate system (LCS $\{x_l, z_l\}$). This Local coordinate system (LCS) continuously rotates and translates with the element, which is defined for every element with the origin fixed at the first node and $x_l$ axis chosen to be the line joining first and second nodes of the element. The $z_l$ axis is chosen to be perpendicular to the $x_l$ axis as shown in Fig. 3.1. The global coordinate system (GCS($x, z$)) is the fixed reference frame.

The motion of an element from the original un-deformed configuration to the actual
deformed configuration can be split into two steps. The first is a rigid rotation and translation of the element. The second step consists of a deformation in the local coordinate system. Assuming that the length of the element is properly chosen, the deformational part of the motion is small relative to the local axes. So the local deformations can be expressed with a low order of non-linearity. The next section describes the beam kinematics.

3.2 Beam Kinematics

Let the coordinates for nodes 1 and 2 of an element in GCS be \((x_1, z_1)\) and \((x_2, z_2)\) as shown in Fig. 3.1. The global nodal displacement vector for an element is defined by

\[
P_g = \begin{bmatrix} u_1 & w_1 & \theta_1 & u_2 & w_2 & \theta_2 \end{bmatrix}^T \tag{3.1}
\]

In Eq. (3.1) \(u\) denotes nodal displacement in the \(x\) direction, \(w\) denotes the nodal displacement in the \(z\) direction and \(\theta\) the nodal rotation, and the subscripts 1 and 2 corresponds to the node 1 and node 2 respectively. The local nodal displacement or deformational displacement vector of an element in LCS is defined by

\[
P_L = \begin{bmatrix} \bar{u} & \bar{\theta}_1 & \bar{\theta}_2 \end{bmatrix}^T \tag{3.2}
\]

Here, \(\bar{u}\), \(\bar{\theta}_1\) and \(\bar{\theta}_2\) in Eq. (3.2) are the changes in the length of the element, deformational rotation at node 1 and node 2 respectively, which are computed using the relations:

\[
\bar{u} = L_n - L_o \tag{3.3}
\]

\[
\bar{\theta}_1 = \theta_1 - \alpha \tag{3.4}
\]

\[
\bar{\theta}_2 = \theta_2 - \alpha \tag{3.5}
\]
The symbols $L_n$ and $L_o$ denote the deformed and undeformed length of the element respectively, which are computed using the relations given below.

$$L_o = \sqrt{(x_2 - x_1)^2 + (z_2 - z_1)^2}$$

$$L_n = \sqrt{(x_2 + w_2 - x_1 - u_1)^2 + (z_2 + w_2 - z_1 - w_1)^2}$$

The angle $\alpha$ in Eqs. (3.4) and (3.5) denotes the rigid-body rotation undergone by the beam element, which is computed using

$$\sin \alpha = c_o s - s_o c$$

$$\cos \alpha = c_o c + s_o s$$
Here, \( c_o \), \( s_o \), \( c \) and \( s \) are given by the following expressions

\[
c_o = \cos \beta_o = \frac{x_2 - x_1}{L_o} \tag{3.10}
\]

\[
s_o = \sin \beta_o = \frac{z_2 - z_1}{L_o} \tag{3.11}
\]

\[
c = \cos \beta = \frac{x_2 + u_2 - x_1 - u_1}{L_n} \tag{3.12}
\]

\[
s = \cos \beta = \frac{z_2 + w_2 - z_1 - w_1}{L_n} \tag{3.13}
\]

### 3.3 Displacement Interpolation

The deformational displacement (\( \hat{u}_m \), \( \hat{w} \) in LCS) within the element interpolated with the shape functions in terms of the nodal deformational displacements is given as

\[
\hat{u}_m = \zeta \hat{u} \tag{3.14}
\]

\[
\hat{w} = L_0 (\zeta (1 - \zeta)^2 \hat{\theta}_1 + \zeta^2 (\zeta - 1) \hat{\theta}_2) \tag{3.15}
\]

where

\[
\zeta = \frac{\hat{x}}{L_0} \tag{3.16}
\]

The symbol \( \hat{u}_m \) denotes the deformation of the neutral axis. The deformation of any point at a distance \( \hat{z} \) from the neutral axis in terms of the \( \hat{u}_m \) and \( \hat{w} \) is given by the following relation:

\[
\hat{u} = \hat{u}_m - \hat{z} \frac{d\hat{w}}{d\hat{x}} \tag{3.17}
\]

The next section describes the strain measure adopted in the present formulation.
3.4 Strain Measure

In geometrically nonlinear analysis the deformational component is usually small as compared with the rigid-body component. Hence, the engineering strain can be employed as a strain measure if it can be expressed in terms of the deformational components. This decomposition is readily achieved with the co-rotational formulation. So the engineering strain measure can be adopted in geometrically nonlinear analysis. The strain in the local co-ordinate system is defined by

\[ \varepsilon = \frac{\partial \hat{u}}{\partial \hat{x}} + \hat{z} \frac{\partial^2 \hat{w}}{\partial \hat{x}^2} \]  

(3.18)

Introducing Eqs. (3.14), (3.15) and (3.17) into Eq. (3.18), the strain can be expressed in terms of the deformational nodal displacements as given below.

\[ \varepsilon = \frac{\bar{u}}{L_o} - \frac{\hat{z}}{L_o} \left( (6\zeta - 4)\bar{\theta}_1 + (6\zeta - 2)\bar{\theta}_2 \right) \]  

(3.19)

By assuming that the material is linearly elastic, the stress and strain are related by

\[ \sigma = E\varepsilon \]  

(3.20)

Where \( E \) is the Young’s modulus. In the next section the expressions for internal forces are presented.

3.5 Internal Forces

The internal forces of an element are obtained from the internal virtual work expression (IVW), which is defined by

\[ IVW = \int_V \sigma \delta \varepsilon \, dv \]  

(3.21)
Here, $\delta \varepsilon_v$ is the virtual strain due to virtual deformational displacement, which is expressed as

$$\delta \varepsilon_v = \frac{\delta \bar{u}}{L_o} + \frac{\ddot{z}}{L_o} \left( (4 - 6 \zeta) \delta \bar{\theta}_1 + (2 - 6 \zeta) \delta \bar{\theta}_2 \right)$$

(3.22)

With the substitution of Eqs. (3.22) and (3.20) into Eq. (3.21), the internal virtual work ($IVW$) becomes

$$IVW = F_L \delta P_L$$

(3.23)

where $F_L$ is the internal force vector in the local coordinate system (LCS) the components of which are expressed as

$$F_L = [N \ M_1 \ M_2]^T$$

(3.24)

$$N = \frac{EA}{L_o}$$

(3.25)

$$M_1 = \frac{EI}{L_o} (4\bar{\theta}_1 + 2\bar{\theta}_2)$$

(3.26)

$$M_2 = \frac{EI}{L_o} (2\bar{\theta}_1 + 4\bar{\theta}_2)$$

(3.27)

The symbols $A$ and $I$ in Eqs. (3.25), (3.26) and (3.27) are the area and moment of inertia of the cross-section of the element respectively. To solve the finite element equilibrium equations it is required to establish the relation between the internal forces in LCS and in GCS.

The conservation of energy is used to get the relation between the internal force ($F_L$) in the local coordinate system and the internal force ($F_g$) in the global coordinate system. That is the internal virtual work in LCS is equal to that in GCS.

$$\delta P_L^T F_L = \delta P_g^T F_g$$

(3.28)

From Eq. (3.28) it is clear that the relation between $F_L$ and $F_g$ can be obtained if we can establish a relation between $\delta P_L$ and $\delta P_g$. This is obtained by expressing the variation
of the local displacement field in terms of the variation of the global displacements.

\[
\delta \bar{u} = \delta L_n \left( \frac{x_2 + u_2 - x_1 - u_1}{L_n} \right) (\delta u_2 - \delta u_1) + \left( \frac{z_2 + w_2 - z_1 - u_1}{L_n} \right) (\delta w_2 - \delta w_1)
\]

\[
= \begin{bmatrix} -\cos \beta & -\sin \beta & 0 & \cos \beta & \sin \beta & 0 \end{bmatrix}^T \delta \mathbf{P}_g
\]

\[
\delta \bar{\theta}_1 = \delta \theta_1 - \delta \alpha
\]

\[
= \delta \theta_1 - \delta \beta
\]

\[
\delta \bar{\theta}_2 = \delta \theta_2 - \delta \alpha
\]

\[
= \delta \theta_2 - \delta \beta
\]

\[
\delta \beta \quad \text{is calculated from Eq. (3.13) or Eq. (3.14)}
\]

\[
\delta \beta = \frac{1}{\cos \beta L_n} \left( (\delta w_2 - \delta w_1) - \sin \beta \cos \beta (\delta u_2 - \delta u_1) - (\sin \beta)^2 (\delta w_2 - \delta w_1) \right)
\]

\[
= \frac{1}{L_n} \begin{bmatrix} -\sin \beta & -\cos \beta & 0 & -\sin \beta & \cos \beta & 0 \end{bmatrix}^T \delta \mathbf{P}_g
\]

From Eqs. (3.29), (3.30) and (3.31), the relation between \( \delta \mathbf{P}_L \) and \( \delta \mathbf{P}_g \) is obtained as

\[
\delta \mathbf{P}_L = \mathbf{B} \delta \mathbf{P}_g \quad \text{(3.33)}
\]

where \( \mathbf{B} \) is given by

\[
\mathbf{B} = \begin{bmatrix}
-\cos \beta & -\sin \beta & 0 & \cos \beta & \sin \beta & 0 \\
-\sin \beta L_n & \cos \beta L_n & 1 & \sin \beta L_n & -\cos \beta L_n & 0 \\
-\sin \beta L_n & \cos \beta L_n & 0 & \sin \beta L_n & -\cos \beta L_n & 1
\end{bmatrix}
\]

Substituting (3.33) into (3.28) the relation between \( \mathbf{F}_L \) and \( \mathbf{F}_g \) is obtained.

\[
\mathbf{F}_g = \mathbf{B}^T \mathbf{F}_L \quad \text{(3.35)}
\]
3.6 Tangent Stiffness Matrix

The global tangent stiffness matrix for an element is obtained by taking the variation of the internal force with respect to the displacement.

\[
\delta F_g = K_g \delta P_g = B^T \delta F_L + N \delta b_1 + M_1 \delta b_2 + M_2 \delta b_3
\]  \hspace{1cm} (3.36)

The first, second and third columns of the matrix \( B^T \) are \( b_1 \), \( b_2 \) and \( b_3 \) in Eq. (3.36) respectively. The following notation is introduced to simplify these relations.

\[
r = [-\cos \beta \quad -\sin \beta \quad 0 \quad \cos \beta \quad \sin \beta \quad 0]^T \hspace{1cm} (3.37)
\]

\[
q = [\sin \beta \quad -\cos \beta \quad 0 \quad -\sin \beta \quad \cos \beta \quad 0]^T \hspace{1cm} (3.38)
\]

The variations of \( r \) and \( q \) are given by

\[
\delta r = q \delta \beta \hspace{1cm} (3.39)
\]

\[
\delta q = -r \delta \beta \hspace{1cm} (3.40)
\]

With this notation \( b_1 \), \( b_2 \) and \( b_3 \) become

\[
b_1 = r \hspace{1cm} (3.41)
\]

\[
b_2 = [0 \quad 0 \quad 1 \quad 0 \quad 0]^T - \frac{1}{L_n} q \hspace{1cm} (3.42)
\]

\[
b_3 = [0 \quad 0 \quad 0 \quad 0 \quad 1]^T - \frac{1}{L_n} q \hspace{1cm} (3.43)
\]
The variations of \( b_1, b_2 \) and \( b_3 \) are given by

\[
\delta b_1 = \delta r = \frac{qq^T}{L_n} \delta P_g \\
\delta b_2 = \delta b_3 = -\frac{\delta q}{L_n} + \frac{q \delta L_n}{L_n^2}
\]

Now, \( \delta F_L \) is computed as

\[
\delta F_L = K_L \delta P_L = K_L B \delta P_g
\]

where \( K_L \) is the tangent stiffness matrix in local coordinate system, which is given by

\[
K_L = \begin{bmatrix}
\frac{EA}{L_n} & 0 & 0 \\
0 & \frac{4EI}{L_n} & \frac{2EI}{L_n} \\
0 & \frac{2EI}{L_n} & \frac{4EI}{L_n}
\end{bmatrix}
\]

So, the expression for the global tangent stiffness for an element becomes

\[
K_g = B^T K_L B + \frac{qq^T}{L_n} N + \frac{1}{L_n^2} (rq^T + qr^T)(M_1 + M_2)
\]

The element force vector \( F_g \) and the element tangent stiffness matrix \( K_g \) are assembled into the global force vector and the tangent stiffness matrix following general finite element assembly procedure. The resulting equations are solved iteratively using Newton-Raphson method.

The strain measure adopted in Sec. 3.4 is prone to membrane-locking. So the strain measure needs to be modified to avoid it. In the next section, we introduce a modified strain measure to avoid membrane-locking.
3.7 Modified strain measure

To overcome the problems associated with membrane locking, there are different approaches in the literature. These methods can be grouped into two categories.

1. Special strategies: In this approach, the strain measure is defined by following the special strategies such that the membrane locking can be avoided.

2. Hybrid finite element methods: In this approach, the strain is interpolated as an extra degree of freedom in addition to the displacement field[23].

In this work, the average strain measure defined by Battini [24] is used.

\[
\epsilon = \frac{1}{L} \int_0^L \left( \frac{\partial \bar{u}}{\partial \bar{x}} + \frac{1}{2} \left( \frac{\partial \bar{w}}{\partial \bar{x}} \right)^2 \right) d\bar{x} - \frac{\bar{z}}{L_o} \bar{\theta}_2^2 - \frac{\bar{\theta}_1^2}{2} + \bar{\theta}_2^2 
\]

(3.49)

3.7.1 Modified internal forces

The following relations are obtained for the internal forces due to the modified strain measure.

\[
N = EA \left\{ \frac{\bar{u}}{L_o} + \frac{1}{15} \left( \bar{\theta}_1^2 - \frac{\bar{\theta}_1 \bar{\theta}_2}{2} + \bar{\theta}_2^2 \right) \right\} 
\]

(3.50)

\[
M_1 = EAL_o \left\{ \frac{\bar{u}}{L_o} + \frac{1}{15} \left( \bar{\theta}_1^2 - \frac{\bar{\theta}_1 \bar{\theta}_2}{2} + \bar{\theta}_2^2 \right) \right\} \left( \frac{2}{15} \bar{\theta}_1 - \frac{1}{30} \bar{\theta}_2 \right) + \frac{EI}{L_o} (4\bar{\theta}_1 + 2\bar{\theta}_2) 
\]

(3.51)

\[
M_2 = EAL_o \left\{ \frac{\bar{u}}{L_o} + \frac{1}{15} \left( \bar{\theta}_1^2 - \frac{\bar{\theta}_1 \bar{\theta}_2}{2} + \bar{\theta}_2^2 \right) \right\} \left( \frac{2}{15} \bar{\theta}_2 - \frac{1}{30} \bar{\theta}_1 \right) + \frac{EI}{L_o} (2\bar{\theta}_1 + 4\bar{\theta}_2) 
\]

(3.52)
3.7.2 Modified local tangent stiffness matrix

The following relations are obtained for the components of the local tangent stiffness matrix due to the modified strain measure.

\[
K_L = \begin{bmatrix}
KL_{11} & KL_{12} & KL_{13} \\
KL_{12} & KL_{22} & KL_{23} \\
KL_{13} & KL_{23} & KL_{33}
\end{bmatrix}
\] (3.53)

\[
KL_{11} = \frac{EA}{L_o} 
\] (3.54)

\[
KL_{12} = EA \left( \frac{2}{15} \bar{\theta}_1 - \frac{1}{30} \bar{\theta}_2 \right) 
\] (3.55)

\[
KL_{13} = EA \left( \frac{2}{15} \bar{\theta}_2 - \frac{1}{30} \bar{\theta}_1 \right) 
\] (3.56)

\[
KL_{22} = EAL_o \left( \frac{2}{15} \bar{\theta}_1 - \frac{1}{30} \bar{\theta}_2 \right)^2 
+ \frac{2}{15} EAL_o \left( \bar{u} \frac{1}{L_o} + \frac{1}{15} \bar{\theta}_1^2 - \frac{1}{30} \bar{\theta}_1 \bar{\theta}_2 + \frac{1}{15} \bar{\theta}_2^2 \right) + \frac{4EI}{L_o} 
\] (3.57)

\[
KL_{23} = EAL_o \left( \frac{2}{15} \bar{\theta}_2 - \frac{1}{30} \bar{\theta}_1 \right) \left( \frac{2}{15} \bar{\theta}_1 - \frac{1}{30} \bar{\theta}_2 \right) 
- \frac{1}{30} EAL_o \left( \bar{u} \frac{1}{L_o} + \frac{1}{15} \bar{\theta}_1^2 - \frac{1}{30} \bar{\theta}_1 \bar{\theta}_2 + \frac{1}{15} \bar{\theta}_2^2 \right) + \frac{2EI}{L_o} 
\] (3.58)

\[
KL_{33} = EAL_o \left( \frac{2}{15} \bar{\theta}_2 - \frac{1}{30} \bar{\theta}_1 \right)^2 
+ \frac{2}{15} EAL_o \left( \bar{u} \frac{1}{L_o} + \frac{1}{15} \bar{\theta}_1^2 - \frac{1}{30} \bar{\theta}_1 \bar{\theta}_2 + \frac{1}{15} \bar{\theta}_2^2 \right) + \frac{4EI}{L_o} 
\] (3.59)

The global tangent stiffness matrix of an element is obtained by substituting Eqs. (3.50), (3.51), (3.52), (3.53) for \( N, M_1, M_2 \) and \( K_L \) in Eq. (3.48).

3.8 Deformation dependent Load

In some applications the external load (such as the drag force, hydrostatic pressure, etc.) acting on the structure is dependent on the deformation of the structure. To model
this kind of loading the finite element method presented so far needs to be modified. This involves accounting for the tangent stiffness terms contributed by the external load, which is clearly observed by linearizing the finite element equilibrium equations.

\[ f_{\text{int}} = f_{\text{ext}} \]  \hspace{1cm} (3.60)

\[ f_{\text{int}}(U_0) + K_{\text{tan}} \Delta U = f_{\text{ext}}(U_0) + K_{\text{ex}} \Delta U \]  \hspace{1cm} (3.61)

Rearrangement of the terms yields

\[ [K_{\text{tan}} - K_{\text{ex}}] \Delta U = [f_{\text{ext}}(U_0) - f_{\text{int}}(U_0)] \]  \hspace{1cm} (3.62)

\[ \Delta U = [K_{\text{tan}} - K_{\text{ex}}]^{-1} [f_{\text{ext}}(U_0) - f_{\text{int}}(U_0)] \]  \hspace{1cm} (3.63)

Here \( K_{\text{tan}} \), \( K_{\text{ex}} \) in Eq. (3.63) are the tangent stiffness terms contributed by the internal force and external forces respectively. \( K_{\text{tan}} \) is computed using eq.(3.48) and \( K_{\text{ex}} \) is computed by linearizing the external force. In this section we derived the additional tangent stiffness terms contributed by the external force considering the follower load and drag force on cylindrical specimen as examples. Derivation of the tangent stiffness matrix due to the follower load is given in Sec. 3.8.1 and due to the drag force on a cylinder is given in Sec. 3.8.2

### 3.8.1 Follower Load Case

The follower load is the special case of loading in which the load at a point rotates with the rotation at that point. Consequently, the nature of the external nodal force depends on the deformation. So there will be contribution of stiffness from the external force. External load in this case is given by

\[ f_{\text{ext}} = Rf_{\text{ex}} \]  \hspace{1cm} (3.64)
Here, $\mathbf{f}_{\text{ex}}$ external load in the undeformed configuration (constant in this formulation) and $\mathbf{R}$ is the rotation matrix. For a two-node beam element $\mathbf{f}_{\text{ex}}$ and $\mathbf{R}$ are given as

$$\mathbf{f}_{\text{ex}} = [f_{x1} \ f_{y1} \ M_1 \ f_{x2} \ f_{y2} \ M_2]^T$$  \hspace{1cm} (3.65)$$

$$\mathbf{R} = \begin{bmatrix}
\cos \theta_1 & -\sin \theta_1 & 0 & 0 & 0 & 0 \\
\sin \theta_1 & \cos \theta_1 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & \cos \theta_2 & -\sin \theta_2 & 0 \\
0 & 0 & 0 & \sin \theta_2 & \cos \theta_2 & 0 \\
0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}$$  \hspace{1cm} (3.66)$$

So, $\mathbf{f}_{\text{ext}}$ is given by

$$\mathbf{f}_{\text{ext}} = \begin{bmatrix}
fx_1\cos \theta_1 - fy_1\sin \theta_1 \\
fx_1\sin \theta_1 + fy_1\cos \theta_1 \\
M_1 \\
fx_2\cos \theta_2 - fy_2\sin \theta_2 \\
fx_2\sin \theta_2 + fy_2\cos \theta_2 \\
M_2
\end{bmatrix}$$  \hspace{1cm} (3.67)$$

Here, $\mathbf{K}_{\text{ex}}$ is given by

$$\mathbf{K}_{\text{ex}} = \frac{d\mathbf{f}_{\text{ext}}}{d\mathbf{U}}$$  \hspace{1cm} (3.68)$$

$$= \begin{bmatrix}
0 & 0 & -(fx_1\sin \theta_1 + fy_1\cos \theta_1) & 0 & 0 & 0 \\
0 & 0 & (fx_1\cos \theta_1 - fy_1\sin \theta_1) & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -(fx_2\sin \theta_2 + fy_2\cos \theta_2) & 0 \\
0 & 0 & 0 & 0 & (fx_2\cos \theta_2 - fy_2\sin \theta_2) & 0 \\
0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}$$  \hspace{1cm} (3.69)$$
3.8.2 Drag on a cylinder

The tangent stiffness matrix derived in the Sec. 3.8.1 is suitable for a follower load. In this section we derive the tangent stiffness due to the drag force on a cylindrical object mounted on a cantilever beam. Figure 3.2 shows an inclined cylinder in a steady wind flow. The drag and lift forces experienced by the cylindrical object with the angle of orientation of the cylinder \( \alpha \) were given by Hoerner [28].

\[
F_D = \frac{1}{2} C_D \rho A V^2
\]

\[
F_L = \frac{1}{2} C_L \rho A V^2
\]

where \( A \) is the frontal area \( (Ld) \), \( V \) the wind velocity and \( C_D \) and \( C_L \) the drag and lift coefficients respectively, which are expressed as functions of \( \alpha \).

\[
C_D = 1.1 \cos^3 \alpha + 0.02
\]

\[
C_L = 1.1 \cos^2 \alpha \sin \alpha
\]
The contribution to the drag and lift forces due to the tip element are more as compared with those of the rest of the beam in the model considered. Hence, we neglect the external force acting on the rest of the beam and focused only on the cylindrical tip. So, the external loading acting on the last element is given by

$$\mathbf{F}_{\text{ext}} = [0 \ 0 \ 0 \ F_D \ F_L \ 0]^T$$

(3.74)

The contribution of the external load to the last element’s tangent stiffness matrix is given by

$$\mathbf{K}_{\text{ex}} = \frac{d \mathbf{F}_{\text{ext}}}{d \mathbf{U}}$$

(3.75)

$$= \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & K_{\text{extd}} & 0 \\ 0 & 0 & 0 & 0 & K_{\text{extl}} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

where $K_{\text{extd}}$ and $K_{\text{extl}}$ are given by

$$K_{\text{extd}} = -\frac{3}{2} \rho A V^2 \cos^2 \alpha \sin \alpha$$

(3.76)

$$K_{\text{extl}} = \frac{1}{2} \rho A V^2 \left( -2 \cos \alpha \sin^2 \alpha + \cos^3 \alpha \right)$$

(3.77)

In the next section the finite element formulation presented in this chapter is validated with benchmark examples.

### 3.9 Validation of the solution procedure

To examine or demonstrate the accuracy of new finite element models or the effectiveness of new nonlinear solution procedures, benchmark problems are often exercised and the
predictions are compared to some reference solutions. The finite element formulation presented in this chapter is validated with various examples such as pure bending of a cantilever beam, cantilever beam with transverse tip load, etc. Some of these examples have analytical solutions.

### 3.9.1 Example 1: Pure bending of a cantilever beam

When a cantilever is subjected to pure bending moment \( M_{\text{max}} = 2\pi EI/L \), it will bend into a circle. This is verified by discretizing cantilever beam into 20 elements. The following parameters were considered for the cantilever beam:

1. Length \( L = 10 \, m \)
2. Width \( b = 1 \, m \)
3. Depth \( d = 0.1 \, m \)
4. Young’s modulus \( E = 1.2e6 \, N/m^2 \)
5. Number of elements = 20

For this data, \( M_{\text{max}} = 62.8319 \). The resulting deformed profiles at the intermediate stages of loading are indicated in Fig. 3.3. The variation of the tip displacement \((u, w)\) with the moment is shown in Fig. 3.4. The analytical solution for this example is given Sec. B.1.
Figure 3.3: Cantilever beam under pure bending moment

Figure 3.4: Tip displacement of cantilever beam under pure bending moment
3.9.2 Example 2: A cantilever beam under transverse tip load

For the data given in Sec. (3.9.1), a cantilever beam under transverse load is verified for $F_{\text{max}} = 5 \, N$, the resulting deformed profile in the intermediate stages of loading is shown in Fig. 3.5 and the tip displacement is compared with the analytical solution [30] in Fig. 3.6. The analytical expression for the tip displacement of the cantilever beam is given in Sec. B.2.

![Deformed profile of the cantilever beam under the transverse tip-load](image1)

**Figure 3.5:** Deformed profile of the cantilever beam under the transverse tip-load

![Tip displacement of the cantilever beam under transverse tip-load](image2)

**Figure 3.6:** Tip displacement of the cantilever beam under transverse tip-load
3.9.3 Example 3: A semicircular arch

In this section, the FEM formulation is validated for semicircular arch problems that were solved with the commercial finite element software ABAQUS. The following parameters were considered in the analysis

1. The radius of the arch = 5 units
2. width = 1 units
3. depth = 0.1 units
4. Young’s modulus $E = 1.2e^6$ units
5. Number of elements = 72

A semi circular arch with a point load at the center

A semi-circular arch subjected to a vertical point load at the center is analysed with ABAQUS and FEM developed in this chapter, the results are compared in Fig. 3.7. The deflected profile at various load steps is shown in Fig. 3.8. In ABAQUS, B22 element was used. As can be seen in the Table 3.1, the two methods give the same result to four decimal places.

![Figure 3.7: Displacement of center point under a point load at the center](image-url)
<table>
<thead>
<tr>
<th>No</th>
<th>Force</th>
<th>FEM W</th>
<th>ABAQUS W</th>
</tr>
</thead>
<tbody>
<tr>
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<td>4</td>
<td>-0.0624</td>
<td>-0.0624</td>
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<tr>
<td>2</td>
<td>8</td>
<td>-0.1333</td>
<td>-0.1333</td>
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<tr>
<td>3</td>
<td>12</td>
<td>-0.2147</td>
<td>-0.2148</td>
</tr>
<tr>
<td>4</td>
<td>16</td>
<td>-0.3093</td>
<td>-0.3094</td>
</tr>
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<td>5</td>
<td>20</td>
<td>-0.4208</td>
<td>-0.4208</td>
</tr>
<tr>
<td>6</td>
<td>24</td>
<td>-0.5544</td>
<td>-0.5546</td>
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<td>7</td>
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<td>-0.7186</td>
<td>-0.7187</td>
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<td>8</td>
<td>32</td>
<td>-0.9283</td>
<td>-0.9282</td>
</tr>
<tr>
<td>9</td>
<td>36</td>
<td>-1.2158</td>
<td>-1.2159</td>
</tr>
<tr>
<td>10</td>
<td>40</td>
<td>-1.6930</td>
<td>-1.6932</td>
</tr>
</tbody>
</table>

Table 3.1: Comparison of the center point displacement of a semicircular arch under a point load

Figure 3.8: Deformed profile of the semicircular arch under a point load
A semi circular arch with an inclined point load at the center

A semi-circular arch subjected to a point load which is acting at angle 45° with the horizontal is analyzed with ABAQUS and FEM developed in this work. The results are compared in Fig. 3.10 and Table 3.2 and the deflected profiles of the arch at various load steps are shown in Fig. 3.9. Once again, the excellent agreement between the two can be seen.

Figure 3.9: Deformed profile of semicircular arch with inclined point load at the center
Figure 3.10: Displacement of center point with inclined load

<table>
<thead>
<tr>
<th>No</th>
<th>Force</th>
<th>FEM</th>
<th>ABAQUS</th>
</tr>
</thead>
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<td></td>
<td>U</td>
<td>W</td>
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<td>-0.0459</td>
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<td>8</td>
<td>-0.1549</td>
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<tr>
<td>3</td>
<td>12</td>
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<td>-0.4965</td>
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<tr>
<td>7</td>
<td>28</td>
<td>-0.7639</td>
<td>-0.6563</td>
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<tr>
<td>8</td>
<td>32</td>
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<td>-0.8488</td>
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<td>9</td>
<td>36</td>
<td>-1.0835</td>
<td>-1.0783</td>
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<tr>
<td>10</td>
<td>40</td>
<td>-1.2477</td>
<td>-1.3504</td>
</tr>
</tbody>
</table>

Table 3.2: Comparison of the center point displacement of a semicircular arch with inclined point load
3.9.4 Example 4: A cantilever beam with a deformation dependent load at the tip

To validate the solution procedure for a deformation-dependent loading case, the following examples are considered.

1. Follower load

2. Drag and lift force due to a cylindrical object at the tip

**Follower load**

The solution obtained from the co-rotational FEA analysis for the follower load is compared with that of ABAQUS. The following numerical data were considered for FEA.

1. Length $L = 10 \text{ m}$
2. Width $b = 1 \text{ m}$
3. Depth $d = 0.1 \text{ m}$
4. Young’s modulus $E = 1.2e6 \text{ N/m}^2$
5. Number of elements = 20

The deformed profiles obtained from the co-rotational FE analysis are shown in Fig. 3.11. Tip displacement obtained from the FEA are compared with ABAQUS solution in Fig. 3.12 and Table. 3.3. Once again, the excellent agreement between the two can be seen.
Figure 3.11: Deformed profile of the cantilever beam when it is loaded with a follower load at the tip

<table>
<thead>
<tr>
<th>No</th>
<th>Force</th>
<th>FEM U</th>
<th>W</th>
<th>ABAQUS U</th>
<th>W</th>
</tr>
</thead>
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<tr>
<td>10</td>
<td>50</td>
<td>-10.2023</td>
<td>4.8025</td>
<td>-10.2033</td>
<td>4.8029</td>
</tr>
</tbody>
</table>

Table 3.3: Comparison of the tip displacement of the cantilever beam when it is subjected to a follower load at the tip
Figure 3.12: Tip displacement of the cantilever beam when it is loaded with a follower load at the tip.
Drag and lift force due to a cylindrical object at the tip

To validate the solution procedure presented in Sec. 3.8.2 for the case of the deformation-dependent load that changes the magnitude with the deformation, (ABAQUS or any other commercial software cannot readily solve this problem) we conducted an experiment. In this experiment the aerodynamic force is applied on the specimen in a wind tunnel experimental setup as shown Fig. 3.13. In this experiment a weightless cylindrical specimen (made of foam) was firmly attached to the tip of the cantilever beam, this kind of arrangement leads to a tip-load on the cantilever, which depends on the rotation of the tip according to the relations given in Eqs. (3.70) and (3.71). In the experiment, the flow rate is adjusted to vary the velocity of the air. In each step the image of the deflected cantilever is captured using a digital camera, which is used for computing the displacement. The deformed and undeformed profiles of the specimen are shown in Fig. 3.14. The tip displacement obtained from the FEA and the experimentally obtained displacement for different wind velocities are shown in the Table. 3.4. The percentage error in the displacement is calculated based on the FEM solution. The error is within 20%. Since the co-rotational finite element formulation implemented in this work gave excellent results for all other benchmark problems, this large error can be attributed due to the inaccuracies in the experiment and the manner in which the displacements were captured.
In this chapter, the implementation details of the co-rotational finite elements are presented. The solution procedure to account for the deformation-dependent external load is considered as an extension to the present co-rotational formulation. The numerical solutions obtained using this implementation are validated with analytical solutions and with the results of commercial software FEA package ABAQUS. An experiment was also conducted to check the accuracy of the deformation-dependent solution in a wind tunnel setup. This co-rotational code is used in two applications in chapter 4 and chapter 5.
Chapter 4

Vision-Based Force-Sensing

4.1 Introduction

In micromanipulation, it is required to observe, position and transform the micron-sized object with minimal damage done to it. For successful manipulation, it is useful to have force feedback. Ideally, a force sensor used for this purpose should be non-intrusive. But most force sensors are intrusive because they interfere with the force being measured. In this chapter, we propose a minimally intrusive method for determining the force from the deformed and undeformed profiles of an elastic member, which are captured visually. This is achieved by constructing the displacement field with the images of elastic member captured in the deformed and undeformed configurations and its material properties. We considered three cases of force sensing schemes in this analysis, the first of which is concerned with the estimation of the magnitude of the force when its location and direction are known. The second is a situation where only the location of the force is known. Finally, we considered a case in which there is no information about the location, the direction or the magnitude of the force. These three schemes of force sensing are validated with numerical experiments (from the displacements obtained from the known force) and with experiments conducted on macro scale prototypes made using spring steel. In the next section we propose a force-sensing method that is suitable for micromanipulation.
4.2 The force-sensing scheme

Towards the application of vision based force-sensing technique in biological studies we propose a scheme consisting of two flexible semicircular arches attached to the microscope stage. These arches support the biological object during manipulation, and the deformations of which help to measure the force. The schematic representation of the force-sensor is shown in Fig. 4.1.

![Figure 4.1: Proposed scheme for vision based force-sensing study in biological studies](image)

During the operation, the semicircular arches are brought together to hold the object by moving the microscope stage. Due to the application of the force on the object semicircular arches get deflected. The displacement field of the arch is computed from the images captured. From this displacement field, force(s) acting at the contact points of the object is/are computed. The unknown external force acting on the object is computed by enforcing the static equilibrium of force.

The force(s) computed using the vision-based force-sensing technique as reported by Wang et al. [8] is susceptible to the measurement error in the displacement. This is due to the spurious forces developed as a result of the error in the displacement field. To overcome this spurious force, we propose two methods of determining the unknown force under the special cases of loading in addition to direct force recovery method from
internal reaction forces. The following cases of loading are considered in this report.

1. The location, direction and magnitude of the force are unknown

2. The location and direction of the force are known

3. The location of the force is known

Geometry and boundary conditions of the structure is known in all the above cases. The loading of the elastic member is assumed to be point load. In next section, we describe the a sensing method suitable to determine the location, direction and magnitude of the force.

4.3 The location, direction and magnitude of the force are unknown

In some applications, the location of the force is also an unknown. To predict the location, direction and magnitude of the force, the direct method of force computation (Wang et al. [8]) is used in this section. In this method, the unknown force is estimated by computing the internal reaction force at all the nodes in the finite element mesh from the displacement field obtained using the image processing data. The algorithm for this, following Wang et al. [8], is shown in Fig. 4.2. This method is highly sensitive to the error in the displacement, so it is not suitable for direct implementation in micromanipulation. Hence, we propose two new force-sensing methods that are capable of determining the magnitude and direction of the force when its location is known. These methods are discussed in Sec. 4.4 and Sec. 4.5.
4.4 The location and the direction of the force are known

In most of the micromanipulation tasks, the location and direction of the force are usually known but magnitude of the force is unknown. Here, a method for determining the unknown magnitude of the force is proposed from the force-deflection relation of an elastic member. Magnitude of the force is computed from the pre-established relation between the force and deflection by performing FE analysis of the structure for a wide range of load magnitudes for the known direction and the location of the force. The procedure to determine the unknown force magnitude is summarized below.

1. Pre-processing

   (a) Extract the displacement at the point of application of the load for a wide range of values of the loading starting from zero to the buckling load of the beam by performing FEA

   (b) Fit a spline (piece-wise polynomial) curve to the load-displacement data.
2. Force computation

(a) From the images of undeformed and deformed configurations of the structure, compute the displacement at the point of application of the load.

(b) Compute the force magnitude from the spline fitted data (pre-processed information).

4.5 The location of the force is known

The force-sensing scheme presented in Sec. 4.4 is limited to determine the magnitude of the force when the location and the direction are known. But in some micro-manipulation activities it is required to determine the direction of force in addition to the magnitude. In this section we propose a force sensing method that is suitable for determining the magnitude and direction of the force, which is explained with the help of fig. 4.3.

![Figure 4.3: Illustration of case 3](image)

To determine the direction of the applied force at a location on the structure (such as point A), it is required to track the displacement of point A and another point B in the neighborhood of A for a wide range of values of the angle (β) the force and its magnitude. This data is used for building a database for the force-deflection relation for every angle of application, which is used for direction computation. In the process of determining the direction, force for all directions is computed using the database at both points A and
$B$ from the displacements measured at the corresponding points. The correct direction among all the directions is the one which gives minimum difference in the force computed at points $A$ and $B$. This procedure is illustrated considering an example of determining the correct direction of loading among two directions with Fig. 4.4. Here, solid lines corresponds to the point $A$ and dashed lines corresponds to the point $B$. To determine the unknown direction of the force (considered direction-2 to test the method), two vertical lines are drawn corresponding to the displacements of the respective points. The dot-dashed vertical line intersects the dot-dashed force-displacement curves at points $P$ and $Q$, where as the solid vertical line intersects the solid force-displacement curves at points $R$ and $S$. The magnitude of the forces corresponding to the points $P$ and $S$ are equal, so the correct direction is direction-2. In this case the forces corresponding to $P$ and $S$ are the same. But it need not be so when displacements of points $A$ and $B$ are not accurately measured. In such cases, we choose the direction that gives the least difference in the two forces.

The procedure to determine the force direction in a general case is summarized.

1. Pre-processing
   
   (a) Extract the displacement at the point of application of the load and at another suitable point in the neighborhood, for a wide range of magnitudes and directions of the force by performing FEA.

   (b) Create a database for the force-deflection relation for each direction.

2. Direction computation

   (a) Compute the magnitudes of the forces corresponding to all directions in the database with the displacements measured (from the images) at two predetermined locations.

   (b) Compute the difference in the magnitudes of the forces for all the directions corresponding to the two points.
(c) Identify the direction of the unknown force as the one having the least difference in the force magnitudes corresponding to the two points.

3. Force computation

(a) Compute the magnitude of the force in the same manner as that of Sec. 4.4 for the direction that is identified as the correct direction.

4.6 Numerical Examples

To demonstrate the accuracy of the predicted force using the sensing methods presented in Sec. 4.4, Sec. 4.5 and Sec. 4.3, we consider some examples in this section. In these
examples the displacement field is obtained from the FEA using the commercial software ABAQUS. With ABAQUS’s displacement field as the input to the force sensing algorithm, force is estimated so that it can be verified to check the accuracy.

4.6.1 Example 1: A semi-circular arch with a point load at the center

In this section we consider a semicircular arch with a point load applied at the center, the details of which are given below.

1. The Radius of the arch = 5 m
2. $EA = 1.2 \times 10^5$ $N$
3. $EI = 100$ $Nm^2$
4. The number of elements = 50

Case 1: Determining the location, direction and magnitude of the force:

In this case the unknown force is determined by computing the nodal internal forces, which requires the displacements of all the nodal points in the finite element mesh. To test the accuracy of force prediction, displacement obtained from the known force is used as the input to the force-sensing algorithm. In this example the displacement data obtained from the vertical point load of 35 $N$ at the center given as input to force prediction algorithm. The predicted force information is shown in Fig. 4.5, in this figure the arrows indicate the direction and relative magnitude of the applied force. The effect of measurement error on force prediction is studied by perturbing the equilibrium displacement field randomly. A random perturbation up to 1% in the displacement field yields the force distribution shown in Fig. 4.6. From this figure it is clear that this method of force prediction is sensitive to the error in measurement, hence it is not suitable for force feedback in micromanipulation. Wang et al. [8] explained the reason for this using sensitivity analysis.
Figure 4.5: Predicted distribution of force with the direct method

Figure 4.6: Predicted distribution of force for 1% error in the displacement field
Case 2: Determining the magnitude of the force:
In this case, we generated the force-deflection characteristic for the point under loading by FEA for the magnitude-range of 0 to 40 N. The spline fitted curve to the force-deflection relation of the loading point and FEA solution are shown in Fig. 4.7. To test the accuracy of the predicted force, the deflection of the center point is determined under a known point load by performing FEA with ABAQUS. With this deflection the input to the spline fitted data, the unknown magnitude of the force is computed. In the present analysis, deflection of the center point of the semicircular arch under the 35 N force is computed using ABAQUS is given as the input to the force-sensing algorithm. The force output of the force-sensing algorithm is shown in Table. 4.1. The effect of error in the measurement of displacement on the force prediction is computed by perturbing the input displacement to the force-sensing algorithm. From the Table. 4.1 it is clear that the effect of error in the measurement of deflection has less impact on the force prediction as compared with direct computation of the force.
Chapter 4. Vision-Based Force-Sensing

### Table 4.1: Predicted magnitude of the force for different perturbations in the displacement

<table>
<thead>
<tr>
<th>No</th>
<th>% Perturbation in displacement</th>
<th>Actual force(N)</th>
<th>Estimated force(N)</th>
<th>% Error in force</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>35</td>
<td>34.993</td>
<td>-0.02</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
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<td>35.694</td>
<td>1.98</td>
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<tr>
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<td>10</td>
<td>35</td>
<td>36.343</td>
<td>3.83</td>
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</tr>
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<td>-5</td>
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<td>34.24</td>
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<td>-20</td>
<td>35</td>
<td>31.627</td>
<td>-9.64</td>
</tr>
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</table>

Case 3: Determining the direction and magnitude of the force:

To determine the direction and magnitude of the force it is required to create the database of force-deflection relation for two suitable points on the structure. In this case we created the database of the force-deflection relation for the points $A$ and $B$ shown in the Fig. 4.8. To check the accuracy of predicted force, the deflection of the points considered in creating the database are determined for the known vertical force applied at the center by FEA using ABAQUS. This displacement given as the input to the force-sensing algorithm so that estimated force can be compared to check the accuracy. In this case a trial force of 35 N is applied at $90^\circ$ with the horizontal, whereas the output of the force sensing algorithm is 34.993 N and direction $90^\circ$. The effect of measurement error in the displacement on force prediction is computed by perturbing the input displacement field to the force sensing algorithm. The percentage error in the predicted force due to the perturbed displacement shown in the Table. 4.2. From this table it is observed that the effect of measurement error in the displacement has almost no effect on the direction and magnitude prediction as compared with direct force recovery method.
Figure 4.8: Points considered in creating the database

<table>
<thead>
<tr>
<th>No</th>
<th>% perturbation in displacement</th>
<th>Actual force(N),β</th>
<th>Estimated force(N),β</th>
<th>% Error force,β</th>
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<td>31.627, 90</td>
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<tr>
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<td>-30</td>
<td>35, 90</td>
<td>29.55, 89</td>
<td>-15.57, -1.11</td>
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</table>

Table 4.2: Predicted magnitude and direction of the force for the perturbation in the displacement
4.6.2 Example 2: A cantilever beam with a point load at the tip

The vision based force sensing methods presented in this chapter are validated with cantilever as the elastic member. Details of the cantilever beam are given below:

1. Length of the beam = 1 m
2. $EA = 1.2e^5$ N
3. $EI = 100 \ Nmm^2$
4. Number of elements = 20

In this example the displacement obtained from a known vertical force (5 N) acting at tip is used for creating the database to force prediction. To test the accuracy of force prediction with cantilever beam, displacement obtained from a trial vertical force of 3 N acting at the tip used as the input to force-sensing algorithm.

**Determining the magnitude of force:**

The unknown magnitude of the force is determined from the spline fit data to the force-deflection relation obtained from the database. The spline fit data and the FEA solution for creating the database used in this example are shown in the Fig. 4.10. The error in the predicted force due to error in displacement is presented in the table. 4.3. The % error in the predicted force is more than twice that in the displacement in all cases.

![Figure 4.9: Spline fit to the force-deflection relation](image-url)
Table 4.3: Predicted magnitude of the force with a cantilever beam as the elastic member

<table>
<thead>
<tr>
<th>No</th>
<th>% Error in displacement</th>
<th>Actual force(N)</th>
<th>Estimated force(N)</th>
<th>% Error in force</th>
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<td>3</td>
<td>2.929</td>
<td>-2.35</td>
</tr>
<tr>
<td>8</td>
<td>-2</td>
<td>3</td>
<td>2.861</td>
<td>-4.64</td>
</tr>
<tr>
<td>9</td>
<td>-3</td>
<td>3</td>
<td>2.795</td>
<td>-6.84</td>
</tr>
<tr>
<td>10</td>
<td>-4</td>
<td>3</td>
<td>2.731</td>
<td>-8.98</td>
</tr>
</tbody>
</table>

Determining the direction and magnitude of the force:

In this example the unknown force magnitude and direction is computed from the data base created according the force sensing scheme explained in the sec. 4.5, database is created for the displacement of the nodes A and B for the vertical force applied at the node A as shown in the Fig.4.10. In this case the displacement data obtained from the trial vertical force (3 N) acting at the tip of the cantilever beam is used for force recovery using the force sensing method presented sec. 4.5. The error in the force recovery due to the error in displacement is presented by perturbing the displacement obtained for the above mentioned load in the table. 4.4. Here too, the error in the predicted force is more than twice that in the displacement. By comparing the performance of the algorithm in this example (cantilever) and that of the previous one (the fixed-fixed arch), an interesting observation can be made. Note that the force-displacement characteristic of the arch is of the geometrically softening type. That is, its slope decreases with the increasing magnitude. Therefore, there is more incremental displacement for a given increment in the force. So, high resolution is possible in prediction for force. The reverse is true in cantilever case, which has geometrical stiffening.
Figure 4.10: Nodes considered for force-displacement data base for determining the direction of force applied with cantilever beam as elastic member

<table>
<thead>
<tr>
<th>No</th>
<th>% perturbation in displacement</th>
<th>Actual force(N), β</th>
<th>Estimated force(N), β</th>
<th>% Error force, β</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>3, 90</td>
<td>3.0004, 90</td>
<td>0.013, 0</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>3, 90</td>
<td>2.9209, 87</td>
<td>-2.64, -3.33</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>3, 90</td>
<td>2.85343, 84</td>
<td>-4.88, -6.66</td>
</tr>
<tr>
<td>4</td>
<td>3</td>
<td>3, 90</td>
<td>2.7962, 81</td>
<td>-6.79, -10</td>
</tr>
<tr>
<td>5</td>
<td>4</td>
<td>3, 90</td>
<td>2.71444, 77</td>
<td>-9.52, -14.44</td>
</tr>
<tr>
<td>6</td>
<td>-1</td>
<td>3, 90</td>
<td>2.9291, 90</td>
<td>-2.3633, 0</td>
</tr>
<tr>
<td>7</td>
<td>-2</td>
<td>3, 90</td>
<td>2.86063, 90</td>
<td>-4.65, 0</td>
</tr>
<tr>
<td>8</td>
<td>-3</td>
<td>3, 90</td>
<td>2.79445, 90</td>
<td>-6.85, 0</td>
</tr>
<tr>
<td>9</td>
<td>-4</td>
<td>3, 90</td>
<td>2.73044, 90</td>
<td>-8.98, 0</td>
</tr>
</tbody>
</table>

Table 4.4: Predicted magnitude and direction of the force with cantilever beam as elastic member
4.7 Experimental verification of vision-based force-sensing

The vision based force-sensing techniques presented in this chapter are experimentally validated with experiment conducted on a centimeter-scale semicircular arch fabricated using spring steel sheet using wire cut electric discharge machining (EDM). To validate the force predicted by the force-sensing algorithm, it is required to compare the predicted force against the known force. In these examples, the specimen is loaded by hanging the weights as shown in Fig. 4.13. Images of the deformed and un-deformed configurations are captured with a digital camera.

4.7.1 A semicircular arch

A simple experiment was conducted with a point load on the semicircular arch fixed at both ends as shown in fig. 4.11. In this figure white dots represent the grid markers used to get the mapping between the deformed and undeformed configurations, which also serve as nodes in the finite element mesh. The nodal point coordinates obtained from the undeformed image using Image-Pro Discovery software [32]. Displacement of any point is obtained from the difference between the deformed and undeformed nodal coordinates. The details of the semicircular arch used in this example are given below.

1. The radius of semicircle = 4 cm
2. The Width = 0.2 mm
3. The thickness = 0.4 mm
4. Young’s Modulus =200 GPa

Magnitude prediction

In this case the unknown magnitude of the force is obtained from the spline fit data to the force-deflection relation for the point of loading obtained from the FEA for a wide range of loading. Displacement of the loading point is obtained from the images is given as input to the force-sensing algorithm. The actual load and recovered load from the algorithm are shown in the Table. 4.5 and in bar graph in Fig. 4.14.
<table>
<thead>
<tr>
<th>No</th>
<th>Applied load (grams)</th>
<th>Recovered load (grams)</th>
<th>%Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.99</td>
<td>0.9786</td>
<td>-1.1518</td>
</tr>
<tr>
<td>2</td>
<td>1.94</td>
<td>1.9193</td>
<td>-1.0685</td>
</tr>
<tr>
<td>3</td>
<td>2.89</td>
<td>2.8243</td>
<td>-2.2732</td>
</tr>
<tr>
<td>4</td>
<td>3.84</td>
<td>3.6957</td>
<td>-3.7579</td>
</tr>
<tr>
<td>5</td>
<td>4.79</td>
<td>4.7404</td>
<td>-1.0345</td>
</tr>
<tr>
<td>6</td>
<td>5.74</td>
<td>5.7310</td>
<td>-0.1573</td>
</tr>
<tr>
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<td>6.69</td>
<td>6.5062</td>
<td>-2.7480</td>
</tr>
<tr>
<td>8</td>
<td>7.64</td>
<td>7.4285</td>
<td>-2.7689</td>
</tr>
<tr>
<td>9</td>
<td>8.59</td>
<td>8.6554</td>
<td>0.7612</td>
</tr>
<tr>
<td>10</td>
<td>9.54</td>
<td>9.4890</td>
<td>-0.5346</td>
</tr>
<tr>
<td>11</td>
<td>10.49</td>
<td>10.4455</td>
<td>-0.4246</td>
</tr>
<tr>
<td>12</td>
<td>11.44</td>
<td>11.5051</td>
<td>0.5688</td>
</tr>
<tr>
<td>13</td>
<td>12.39</td>
<td>12.3684</td>
<td>-0.1742</td>
</tr>
<tr>
<td>14</td>
<td>13.34</td>
<td>13.5917</td>
<td>1.8869</td>
</tr>
<tr>
<td>15</td>
<td>14.29</td>
<td>14.1096</td>
<td>-1.2625</td>
</tr>
<tr>
<td>16</td>
<td>16.62</td>
<td>16.3748</td>
<td>-1.4753</td>
</tr>
<tr>
<td>17</td>
<td>18.95</td>
<td>18.7448</td>
<td>-1.0828</td>
</tr>
<tr>
<td>18</td>
<td>21.29</td>
<td>21.1028</td>
<td>-0.8794</td>
</tr>
<tr>
<td>19</td>
<td>23.64</td>
<td>23.6789</td>
<td>0.1647</td>
</tr>
<tr>
<td>20</td>
<td>25.99</td>
<td>26.3319</td>
<td>1.3157</td>
</tr>
</tbody>
</table>

Table 4.5: Applied load and recovered load from the vision based force-sensing algorithm with semicircular arch as the elastic member
Magnitude and direction recovery

In this case magnitude and direction of the unknown force are determined by using the force sensing method presented in Sec. 4.5. To determine the direction and magnitude of the force, it is required to create the data base of the two point displacements and the applied force. For this example we considered the displacement of points $A$ and $B$ shown in Fig. 4.13. The performance force sensing method evaluated with the displacements of the points $A$ and $B$ obtained from the images capture during actual loading of the specimen for the loads indicated in Table 4.6 and in the bar graph shown in Fig. 4.15.
<table>
<thead>
<tr>
<th>No</th>
<th>Applied load (grams), $\beta$</th>
<th>Recovered load (grams), $\beta$</th>
<th>% Error force, $\beta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.99, 90</td>
<td>0.9786, 90</td>
<td>-1.1518, 0</td>
</tr>
<tr>
<td>2</td>
<td>1.94, 90</td>
<td>1.9193, 90</td>
<td>-1.0685, 0</td>
</tr>
<tr>
<td>3</td>
<td>2.89, 90</td>
<td>2.8243, 90</td>
<td>-2.2732, 0</td>
</tr>
<tr>
<td>4</td>
<td>3.84, 90</td>
<td>3.6957, 90</td>
<td>-3.7579, 0</td>
</tr>
<tr>
<td>5</td>
<td>4.79, 90</td>
<td>4.7404, 90</td>
<td>-1.0345, 0</td>
</tr>
<tr>
<td>6</td>
<td>5.74, 90</td>
<td>5.7310, 90</td>
<td>-0.1573, 0</td>
</tr>
<tr>
<td>7</td>
<td>6.69, 90</td>
<td>6.5062, 90</td>
<td>-2.7480, 0</td>
</tr>
<tr>
<td>8</td>
<td>7.64, 90</td>
<td>7.4285, 90</td>
<td>-2.7689, 0</td>
</tr>
<tr>
<td>9</td>
<td>8.59, 90</td>
<td>8.6554, 90</td>
<td>0.7612, 0</td>
</tr>
<tr>
<td>10</td>
<td>9.54, 90</td>
<td>9.4890, 90</td>
<td>-0.5346, 0</td>
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<td>11</td>
<td>10.49, 90</td>
<td>10.4455, 90</td>
<td>-0.4246, 0</td>
</tr>
<tr>
<td>12</td>
<td>11.44, 90</td>
<td>11.5051, 90</td>
<td>0.5688, 0</td>
</tr>
<tr>
<td>13</td>
<td>12.39, 90</td>
<td>12.3684, 90</td>
<td>-0.1742, 0</td>
</tr>
<tr>
<td>14</td>
<td>13.34, 90</td>
<td>13.5917, 90</td>
<td>1.8869, 0</td>
</tr>
<tr>
<td>15</td>
<td>14.29, 90</td>
<td>14.1096, 90</td>
<td>-1.2625, 0</td>
</tr>
<tr>
<td>16</td>
<td>16.62, 90</td>
<td>16.3748, 90</td>
<td>-1.4753, 0</td>
</tr>
<tr>
<td>17</td>
<td>18.95, 90</td>
<td>18.7448, 90</td>
<td>-1.0828, 0</td>
</tr>
<tr>
<td>18</td>
<td>21.29, 90</td>
<td>21.1028, 90</td>
<td>-0.8794, 0</td>
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<td>19</td>
<td>23.64, 90</td>
<td>23.6789, 90</td>
<td>0.1647, 0</td>
</tr>
<tr>
<td>20</td>
<td>25.99, 90</td>
<td>26.3319, 90</td>
<td>1.3157, 0</td>
</tr>
</tbody>
</table>

Table 4.6: Applied and recovered magnitude and direction of the load from vision based force sensing with semicircular arch
Figure 4.13: Comparison of the deformed profile of semicircular arch under 14.29 g load

Figure 4.14: Applied load and recovered load from the vision based force-sensing algorithm with semicircular arch as the elastic member
Figure 4.15: Applied and recovered magnitude of the load from the vision based force sensing algorithm with scheme-2 using semicircular arch as elastic member
4.8 Closure

In this chapter, the vision based force-sensing technique explored towards the application of force feedback in micromanipulation. Two novel force-sensing sensing methods are implemented to determine the force when the location is known. The accuracy of the force prediction with these force-sensing schemes are presented with the numerical experiments and real-time experiments on a centimeter-scale model. Prototypes made of spring steel machined using the wire EDM are used in these experiments. The results indicate that it is a promising method for force-sensing in micromanipulation applications.
Chapter 5

Size Optimization of Plant Stems

5.1 Introduction

Wind-related crop damage is a major obstacle to cereal production (mainly in wheat) that costs several billion dollars per year [10]. Mechanical damage to the wheat plant is due to wind-induced stress [11], resonance [12], anchorage rotation and buckling [10]. These biomechanical factors play an important role in the growth and endurance of wheat plants. Hence, the plant biomechanics is an important area of study. While stress, resonance and buckling are familiar terms to engineers, “anchorage rotation” is a new term. It refers to one of the main modes of failure wherein the plant is uprooted at its ground anchor. If we model the vertically growing plant’s stem as a cantilever beam, then the anchorage rotation depends on the reaction moment at the ground anchor. Minimizing this moment helps in preventing the anchorage rotation. Adequate strength and stiffness are also additional criteria while the mass of material of the stem is limited. Thus, optimization seems to be inherent in plant stems.

From the viewpoint of structural optimization, this is an interesting problem in two ways. First, the wind loads acting on the plant stems are deformation-dependent. This is because significant wind load acts primarily on the grain/flower bearing spike at the tip. This load changes as the orientation of the spike changes due to the deformation caused by the wind load itself (see Fig.5.1). So, the load here is dependent on the deformation,
a situation that is rarely considered in structural optimization. It is important to notice that this is clearly different from design-dependent loads [26]. The second interesting aspect of this problem is that there is a conflict between compliant and stiff designs in plant stems.

The actual loading pattern on the wheat-plant is not known exactly. So, we consider two kinds of loading in this report. In the first case, we assumed that, the loading on the plant is varies according to the Eq. (5.1).

\[ F = \frac{1}{2} C_D \rho A_f V^2 \]

\[ = F_0 \cos \theta \]  

(5.1)

From the above equation we can see that increased deformation help to reduce the tip load. Where \( F \) is the aerodynamic load on an elliptical shape oriented at an angle \( \theta \) with the direction perpendicular to that of the air-flow velocity \( V \). Here, drag coefficient \( C_D \), frontal area \( A_f \), mass density \( \rho \) and \( V \) are combined into a constant term \( F_0 \) and \( \cos \theta \) term that includes the effect of orientation.
In the second case, we idealize the spike at the tip of the plant as the cylinder. The variation of the drag and lift force on the cylinder with its orientation [28] are given in Eqs. (5.2) and (5.3).

\[ F_D = \frac{1}{2} C_D \rho AV^2 \]  
\[ F_L = \frac{1}{2} C_L \rho AV^2 \]

where \( A \) is the frontal area, \( V \) the wind velocity and \( C_D \) and \( C_L \) are the drag and lift coefficients respectively. The \( C_D \) and \( C_L \) are the function of the orientation \( \theta \).

\[ C_D = 1.1 \cos^3 \theta + 0.02 \]  
\[ C_L = 1.1 \cos^2 \theta \sin \theta \]

From the Eqs. (5.1), (5.2) and (5.3) it is clear that, as the plant deforms and bends more, not only the tip load reduces but also the moment arm relative to the ground anchor reduces. Consequently, the reaction moment reduces. Thus, there are two motivations for preferring compliant stems. But excessive deformation has other problems. These are: breaking of the stem due to excessive stress; excessive deformation that makes plants hit each other; resonant behavior under varying wind loads; and finally buckling under the self-weight. Thus, plant’s probably seek optimal profiles for adequate stiffness, strength against buckling and material failure and resonance while preventing anchorage rotation due to the reaction moment felt at the ground anchor. There is also the resource condition of available mass as the plant grows and sustains itself.

In this report, we consider the size-optimization of plans stem by modeling it as a cantilever beam including the large deformation effects. As noted above, the load depends on the deformation which is taken into account in the geometrically nonlinear finite element analysis with co-rotational beam elements presented in chapter 3. Due to this kind of loading pattern, there will be an additional contribution to the tangent stiffness matrix due to the external load. For simplicity and due to the lack of information, we
consider only linearly elastic material behavior. The properties of wheat plants stems are taken from [10]. In addition to the load on the spike, there exists distributed load all along the stem and leaves but it is negligible when compared with the load on the spike.

The problem is posed in different ways to understand the most suitable profile of the area of cross-section of the plant along its stem. Prevention of the anchorage rotation is taken as the primary criterion. Stiffness and strength are taken as secondary criteria. Buckling and resonance are not considered in this report.

5.2 Problem statement

Since uprooting at the ground anchor due to large reaction moment is the most significant failure mode in wheat plants [10], we first pose the following problem.

\[
\begin{align*}
\text{Minimise} & \quad M_R \\
\text{Subject to} & \quad \text{Equilibrium equations} \\
& \quad d_l \leq d(x) \leq d_u
\end{align*}
\] (5.6)

where the diameter of the circular cross-section is denoted by \(d(x)\) as the diameter along the length of the beam. But this is an ill-posed problem as the stem becomes as flexible as it can, limited only by the lower bound \(d_l\), in order to deform to achieve \(\theta \to \frac{\pi}{2}\) to make the force zero as per Eq. (5.1). Clearly, the stress in that case may exceed the strength and the stiffness is compromised too. The strength consideration can be added in two ways. The first one is given below.
Chapter 5. Size Optimization of Plant Stems

\[
\text{Minimise } M_R \quad (5.7)
\]

Subject to

\[
\begin{align*}
\text{Equilibrium equations} \\
\max(\sigma(x)) & \leq S \\
d_l & \leq d(x) \leq d_u
\end{align*}
\]

where \(\sigma\) is the stress and \(S\) the permissible strength. In the second method, we pose the strength as the primary criterion while trying to lower the reaction moment at the anchor indirectly. This leads to the formulation shown below.

\[
\text{Minimise } \{S - \max(\sigma(x))\}^2 \quad (5.8)
\]

Subject to

\[
\begin{align*}
\text{Equilibrium equations} \\
\int_0^L \frac{\pi}{4} d^2 dx & \leq V^* \\
d_l & \leq d(x) \leq d_u
\end{align*}
\]

where \(V^*\) is the permitted maximum volume.

In the problems considered above there is no constraint imposed on the displacement, but it is required to limit the horizontal deflection of the tip to prevent the mechanical damage to the plant. The optimization problem is modified to account for the constraint on the horizontal deflection with the formulation presented below.

\[
\text{Minimise } M_R \quad (5.9)
\]

Subject to

\[
\begin{align*}
\text{Equilibrium equations} \\
\max(\sigma(x)) & \leq S \\
U_{x=L} & \leq \Delta \\
d_l & \leq d(x) \leq d_u
\end{align*}
\]

where \(\Delta\) is the permitted maximum deflection of the tip.
5.3 Results

The optimization problems stated in the previous section are solved with the `fmincon` optimizer routine available in the MATLAB optimization tool box. The finite difference gradients were used. The following numerical data were used in the optimization process.

Youngs modulus (E) = 0.5 GPa
Height of the plant (L) = 1 m
Tip load \((F_0) = 10 \text{ N}\)

5.3.1 Case 1

In this section, the analysis was carried out with the assumption that the tip load varies according to the Eq. (5.1).

The optimum cross-section profile obtained for the problem stated in Eq. (5.7) with \(V^* = \pi d_u^2 L/12\) is shown in Fig. 5.2. In this example \(d_l = 2 \text{ mm}\) and \(d_u = 6 \text{ mm}\) are adopted for the lower and the upper bounds on the diameter of the stem respectively.

![Figure 5.2: Optimum solution obtained for the problem stated in Eq. (5.7)](image)

The optimum cross-section profile obtained for the problem stated in Eq. (5.8) with lower bound \(d_l = 2 \text{ mm}\) and upper bound \(d_u = 6 \text{ mm}\) is shown in Fig. 5.3. The tip deflection \(\Delta\) for the various values of \(S\) is indicated in Fig. 5.3.

It can be noticed in Fig. 5.3 that as the value of \(S\) increases, there is a tendency towards compliant designs. In all cases, the reaction moment is less than \(F_0 L\) and it gets better with increasing values of \(S\). The horizontal deflection in the last case with
Figure 5.3: Optimum solution obtained for the problem stated in Eq. (5.8). (a) $S = 2\, MPa$, $\Delta = 0.0590 \, m$, (b) $S = 3\, MPa$, $\Delta = 0.0591 \, m$, (c) $S = 6\, MPa$, $\Delta = 0.0597 \, m$, (d) $S = 12\, MPa$, $\Delta = 0.0895 \, m$, (e) $S = 15\, MPa$, $\Delta = 0.1146 \, m$. Volume constraint is active only in case (a).

$S = 15 \, MPa$ is about 11.5% of the height of the plant. The solution shown in Fig. 5.2 had a horizontal deflection of more than 33% of the height of the plant. Thus, it is important to impose an upper bound on the deflection. This takes care of the stiffness requirement, which is also important for preventing the mechanical damage to the plant. This is included in the problem statement given in Eq. (5.9).

Figure 5.4 and Fig. 5.5 show the sample results of problem stated in Eq. (5.9) without and with strength constraint respectively. It can be seen that the optimum profiles are those that limit the deflection to the upper portion so that it can deform and lower the tip-load as per Eq. 5.1 while retaining stiffness and ensuring that the reaction moment is minimized. It should also be noticed that many elements have reached the upper or lower bounds in both cases.
Figure 5.4: Optimium solution obtained for the problem stated in Eq. (5.9) with the constraint on the tip deflection \( \frac{\Delta L}{L} = 0.2 \).

Figure 5.5: Optimium solution obtained for the problem stated in Eq. (5.9) with the constraint on the tip deflection \( \frac{\Delta L}{L} = 0.2 \) and constraint on the strength \( S = 15 \text{ MPa} \).
5.3.2 Case 2

In this section, the analysis was carried out with the assumption that the tip load varies according to the Eq. (5.2). The lift component of the force is neglected in the present analysis.

The optimum cross-section profile obtained for the problem stated in Eq. (5.9) with the constraint on the tip deflection is shown in Fig. (5.6). In this example \( d_l = 2.5 \text{ mm} \) and \( d_u = 10 \text{ mm} \) are adopted for the lower and the upper bounds on the diameter of the stem respectively.

![Figure 5.6: Optimium solution obtained for the problem stated in Eq. (5.9) with the constraint on the tip deflection \( \Delta L = 0.3 \) for the loading Case 2](image)

5.4 Closure

Biomechanical factors are very important for cereal crop plants such as wheat to prevent uprooting due to ground reaction moment, stress, buckling, etc., when they are subject to wind loads. In this report we consider only the steady wind loads. As explained in this report, the determination of the cross-section profile of the plant stem (modelled here as a cantilever beam) leads to a novel structural optimization problem with deformation-dependent loads. Another interesting aspect is formulating different optimization problems to see why plants prefer compliant stems. Competing effects of minimizing reaction moment at the ground anchor, maximum stress and adequate stiffness are considered here.
Chapter 6

Concluding Remarks

The work presented in this report was motivated by the need to manipulate and mechanically characterize single biological cells. Two aspects of this long-term goal are addressed in this work. First, an inexpensive chemical etching of glass capillaries is accomplished. The etched capillaries enable easy capturing of cells using aspiration. Second, by noting that force feedback is important for manipulation, the vision-based noninvasive force-sensing method reported in the earlier work is extended further. The key component of the work is the implementation of the co-rotational formulation for the geometrically nonlinear finite element analysis of frames. The code developed for the purpose of vision-based force-sensing was also applied to another biology problem of interest. It deals with compliant stems of cereal plants.

6.1 Contribution

The contributions of the work presented in this report are summarized below.

1. Co-rotational finite element analysis was successfully implemented for geometrically nonlinear elastic analysis of frames.

2. Two new numerical algorithms for vision-based force-sensing are developed to determine the magnitude and direction of the force when the location is known. These
methods have the potential applications for force feedback in micromanipulation and in microassembly where the location of the force is usually known.

3. To understand the compliance structure of the plant-stems, an investigation is carried out by posing it as a size optimization problem. In this problem, the plant stem is modeled as a cantilever beam with deformation-dependent load at the tip to minimize the reaction moment at the ground support. The optimization result showed that a compliant design is preferred over the stiff design for this purpose.

6.2 Potential extensions

The work presented in this report can be extended in the following ways.

1. *Vision based force feed-back for micromanipulation*

   - Once the experimental setup is available for handling the biological cells, the numerical algorithms developed in this work can be immediately applied to estimate the forces involved in the manipulation of the cells.

   - The force-sensing method for micromanipulation can be automated with minimal manual intervention if the image processing algorithms are integrated with the force-sensing algorithm.

2. *Plant mechanics*

   - To understand the realistic behavior of the plants it is required to account for the material nonlinearity and lift force in the FE analysis used by the optimization algorithm.

   - The effects of buckling behavior and non-linear dynamic behavior need to be accounted in order to identify other factors that are responsible for the compliance of the stems.
Appendix A

Fabrication of Micro Injector

In this appendix, we present the surface tension controlled etching technique for fabrication of tips of microcapillaries. Chemical etching of glass capillaries was done with an experimental setup consisting of an etchant bath and a stand for holding the tube that is held by a string. Profiles of the etched glass tubes are obtained with Image-Pro discovery software. Comparison of theoretical and experimental studies are presented. In the next section, the principle of the tip-formation with chemical etching technique is presented.

A.1 The principle of the tip-formation

Figure A.1 illustrates the formation of a sharp tip for the micro probe from a capillary glass tube. When glass capillary tube is immersed into an etchant bath with a layer of organic solvent atop the etchant, it wets the surface of the capillary and forms the meniscus as shown in Fig. A.1(a). As the time proceeds, etching of the glass capillary tubing takes place resulting in the decrease of the meniscus height due to the reduction in the outer diameter of the capillary tube as shown in Fig. A.1(b). This process continues until the meniscus reaches the center of capillary and resulting in a sharp tip as shown in Fig. A.1(c). The tip geometry is governed by the meniscus profile, which is obtained by solving the Young-Laplace equation. The Young-Laplace equation in axisymmetric
Figure A.1: Schematic illustration of formation of microinjector tip a). Immersed glass capillary tube b). Glass capillary during etching c). Fully etched glass capillary case [25] can be written as

$$\frac{d^2 y}{dx^2} = \left[1 + \left(\frac{dy}{dx}\right)^2\right] \left(\frac{\Delta \rho g y}{\sigma} \left[1 + \left(\frac{dy}{dx}\right)^2\right]^{1/2} - \frac{1}{x} \frac{dy}{dx}\right)$$  \hspace{1cm} (A.1)

$$\frac{dy}{dx} = -\tan\phi \quad \text{at} \quad x = r_o$$  \hspace{1cm} (A.2)

$$y \to 0 \quad \text{as} \quad x \to \infty$$  \hspace{1cm} (A.3)

where

- $\rho$ the mass density
- $g$ the acceleration due to gravity
- $y$ the meniscus height at a distance $x$ from the center of the cylinder

With the following conditions

Asymptotic solution for Young-Laplace Eq. (A.1) subject to the boundary conditions Eqs. (A.2) and (A.3) as given by Lo [25] is as follows.

$$Z(r) = -c \ln \epsilon + c (\ln 4 - \gamma) - c \ln \left(r + (r^2 - c^2)\right)$$  \hspace{1cm} (A.4)
if \( r = 1 \)

\[
Z = \sin \phi \ln \left( \frac{4}{\epsilon (1 + \cos \phi)} - \gamma \right)
\]  

(A.5)

Where,

- \( c = \sin \phi \).
- \( \phi \) the angle between meniscus and x-axis,
- \( r_o \) the outer radius of the glass tube,
- \( Z = \frac{u}{r_o} \),
- \( r = \frac{x}{r_o} \),
- \( \frac{r_o}{L_o} \) the perturbation parameter (\( \epsilon \))
- \( L_o = \sqrt{\frac{\pi}{\rho g}} \),
- \( \gamma \) is the Euler constant

Meniscus profiles obtained from the asymptotic solution are shown in the Fig. A.3. The final tip profile is obtained by joining the upper ends of the meniscus profiles. The tip profile obtained in this example is shown in Fig. A.4.
A.2 The Experimental Procedure

Micro injectors are fabricated by chemically etching the capillary glass tubing with the experimental setup shown in Fig. A.5. In this setup, the glass tube is held by hanging with a thread from a stand. The position of the glass tube in the etchant bath is adjusted by varying the length of the thread. The step-by-step procedure for etching of capillary glass tubing is described below.

1. **Prepare the glass tube for etching.**

   The coating on the outer surface of the capillary is removed from one end of the capillary up to a length of 2 cm. This step is required, only if the tube is coated. This is usually carried out by chemically with hot piranha or thermally by an open
2. Fill the glass tube with an organic solvent.

The inner geometry of the glass tube is protected from the etchant by filling it with the organic solvent such as silicone oil. Silicone oil is filled into the glass tube by dipping it into the silicone oil bath, due to the capillary action silicone oil rises into tube and fill it. With this arrangement, HF will etch only the outer surface of the capillary.

3. Dip the capillary tube in the etchant bath.

Now the silicone oil filled capillary is dipped into HF (48%) solution with a layer of silicone oil (organic solvent) atop the solution. The capillary tube is immersed in such a way that the HF/oil interface on the innersurface of the capillary is much lower than that of the contact line on the outersurface.

4. Clean the glass capillary tube after etching.
The etched tube from the etchant bath is removed after completion of the tip formation. Followed by this, the tube is thoroughly cleaned in water to remove any traces of HF in it. This step preserves the tip-geometry during usage.
Appendix A. Fabrication of Micro Injector

A.3 Extracting the etched profile data

The X and Y coordinates of the etched profile of the tip obtained using *Image-Pro discovery* [32] (image processing software) from the image of the etched capillary. Due to the symmetry of the glass tube, it is sufficient to compare the tip profile of one edge. The procedure to obtain X and Y coordinates of the tip-profile is summarized below.

1. The image of the etched capillary is loaded into the active window.

2. Magnification factor of the image is selected from spatial calibration option which contains the pre-calibrated reference data for various magnifications.

3. Now point-wise data is obtained from manual measurement menu with *create point* feature option shown in Fig. A.6

4. This data is exported to an excel file with the *Input/output* option available in the measurement menu.

Figure A.6: Extracting etched profile data of quartz glass tube I.D. 0.3 mm and O.D 1 mm
A.4 Experimental Results

Micro probes fabricated by chemically etching the following capillary tubes.

1. *Borosilicate glass tubing of length 10 cm*

   (a) Inner diameter 0.5 mm
       Outer diameter 1.0 mm

   (b) Inner diameter 0.75 mm
       Outer diameter 1.0 mm

2. *Quartz glass tubing*

   (a) Inner diameter 0.3 mm
       Outer diameter 1.0 mm
       Length 7.5 cm

The tip profiles of the etched capillaries obtained with CCD camera and Olympus Microscope (shown in Fig. A.7). The etched profiles of the capillaries are shown in Figs. A.8, A.9 and A.10.
Figure A.7: Olympus microscope

Figure A.8: Etched profile of borosilicate glass tube I.D. 0.5 mm and O.D 1 mm
Figure A.9: Etched profile of borosilicate glass tube ID. 0.75 mm and OD 1 mm

Figure A.10: Etched profile of quartz glass tube ID. 0.3 mm and OD 1 mm
A.5 Comparison of the tip profiles

The tip profile obtained by solving the Young-Laplace equation Eq. (A.1) and experimentally obtained profiles are shown in Fig. A.11 and Fig. A.12.

![Figure A.11: Comparison between the experimental and asymptotic tip profile for ID 0.5 mm and O.D 1 mm](image1)

![Figure A.12: Comparison between the experimental and asymptotic tip profile for ID 0.75 mm and O.D 1 mm](image2)
Appendix B

Analytical solutions for large deflections of beams

In this appendix, analytical solutions for a cantilever beam subjected to the following cases of loading are presented.

1. A cantilever beam subjected to a pure bending moment at the tip

2. A cantilever beam subjected to a transverse load at the tip

B.1 A cantilever beam subjected to a pure bending moment at the tip

The cantilever shown in Fig. B.1 is fixed at A and it is subjected a bending moment of $M$ at the tip. The length of the beam is $L$ and its flexural rigidity is $EI$. In this derivation origin is taken at $A$. The bending moment at a point $Q(x, y)$ is given by

$$EI \frac{d\psi}{ds} = -M \quad (B.1)$$

$$\frac{d\psi}{ds} = \frac{M}{EI} \quad (B.2)$$

$$ds = \frac{EI}{M} d\psi \quad (B.3)$$
Figure B.1: A cantilever beam with a pure bending moment applied at the tip

Integrating the above equation, we will get

\[ \int_0^L ds = \frac{EI}{M} \int_0^{\psi_o} d\psi \] \hspace{1cm} (B.4)

\[ \psi_o = \frac{ML}{EI} \] \hspace{1cm} (B.5)

The x and y co-ordinates of the deformed profile of the cantilever is obtained by using the following relations.

\[ dx = ds \cos \psi \] \hspace{1cm} (B.6)

\[ dy = ds \sin \psi \] \hspace{1cm} (B.7)

Integrating the above equations we will get the co-ordinates of the deformed profile

\[ \int_0^x dx = \frac{EI}{M} \int_0^\psi \cos \psi \, d\psi \] \hspace{1cm} (B.8)

\[ x = \frac{EI}{M} \sin \psi \] \hspace{1cm} (B.9)

\[ \int_0^x dy = \frac{EI}{M} \int_0^\psi \sin \psi \, d\psi \] \hspace{1cm} (B.10)

\[ y = \frac{EI}{M} (1 - \cos \psi) \] \hspace{1cm} (B.11)
The tip displacement of the cantilever is obtained by substituting $\psi = \psi_0$ in Eqs. B.9 and B.11

\begin{align*}
u_{\text{tip}} &= L - \frac{EI}{M} \sin \psi_0 \\
w_{\text{tip}} &= \frac{EI}{M} (1 - \cos \psi_0)
\end{align*}

(B.12) \quad (B.13)

B.2 A cantilever beam subjected to a transverse load at the tip

In this section, analytical solution for the large deflection analysis of the cantilever beam subjected to a point load at the tip is presented. The cantilever beam in Fig. B.2 is fixed at $A$ and it is subjected a point load $F$ at the tip. The length of the beam is $L$ and its flexural rigidity is $EI$. In this derivation the origin is taken at $A$. The bending moment at a point $Q(x, y)$ is given by

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{cantilever_beam.png}
\caption{A cantilever beam with a point load applied at the tip}
\end{figure}

\begin{align*}
M &= EI \frac{d\psi}{ds} \\
&= F(L - x - u_{\text{tip}})
\end{align*}

(B.14)
Differentiating Eq. (B.14) with respect to $s$

\[
\frac{d^2\psi}{ds^2} = -\frac{F}{EI} \frac{dx}{ds}
= -\frac{F}{EI} \cos\psi
\]  

(B.15)

Integrating the above equation will results in

\[
\frac{1}{2} \left(\frac{d\psi}{ds}\right)^2 = -\frac{F}{EI} \sin\psi + C
\]  

(B.16)

The differentiation of Eq. B.14 and integration of the result was carried out in order to establish a differential equation containing the one dependent variable only. The integration constant $C$ can be found from the boundary conditions. In this case, the curvature of the beam at the loading point is considered. If the slope at the loading point is $\psi_o$, than we have

\[
\left(\frac{d\psi}{ds}\right)_{\psi=\psi_o} = 0
\]  

(B.17)

hence,

\[
C = \frac{F}{EI} \sin\psi_o
\]  

(B.18)

Substituting $C$ into Eq. B.16 will yield

\[
\frac{d\psi}{ds} = \frac{2F}{EI} \sqrt{(\sin\psi_o - \sin\psi)}
\]  

(B.19)

By assuming that, the beam as an inextensible member.

\[
\int_0^{\psi_o} ds = L
\]  

(B.20)
with $\frac{FE_0}{ET} = q^2$ and combining Eqs. (B.19) and (B.20) the following expression can be obtained

$$\int_{\psi_0}^{\psi_o} ds = L \quad \text{(B.21)}$$

$$= \frac{L}{q} \int_{\psi_0}^{\psi_o} d\psi \frac{d\psi}{\sqrt{2(\sin \psi_0 - \sin \psi)}}$$

$$q\sqrt{2} = \int_{\psi_0}^{\psi_o} d\psi \frac{d\psi}{\sqrt{(\sin \psi_0 - \sin \psi)}} \quad \text{(B.22)}$$

In order to bring the right side of the Eq. (B.22) to the standard form of elliptical integrals new variables $\phi$ and $p$ are introduced. These variables chosen such that, they will satisfy the following relations.

$$1 + \sin \psi = (1 + \sin \psi_o) \sin^2 \phi \quad \text{(B.23)}$$

$$p^2 = \frac{(1 + \sin \psi_o)}{2} \quad \text{(B.24)}$$

Differentiating the above equation with respect to $\phi$ will results in

$$\cos \psi \frac{d\psi}{d\phi} = 4p^2 \sin \phi \cos \phi \quad \text{(B.25)}$$

But

$$\cos \psi = \sqrt{(1 - \sin^2 \psi)} \quad \text{(B.26)}$$

$$= \sqrt{4p^2 \sin^2 \phi - 4p^4 \sin^4 \phi}$$

$$= 2p \sin \phi \sqrt{(1 - p^2 \sin^2 \phi)}$$

The following relations obtained for $\sin \psi$ and $\sin \psi_o$ in terms of $p$ and $\phi$.

$$\sin \psi = 2p^2 \sin^2 \phi - 1 \quad \text{(B.27)}$$

$$\sin \psi_o = 2p^2 - 1 \quad \text{(B.28)}$$
Substituting for $d\psi$, $\sin \psi$ and $\sin \psi_o$ in Eq. (B.22) we obtain the following expression.

$$q \sqrt{2} = \int_0^{\psi_o} \frac{\sqrt{2} \, d\phi}{\sqrt{(1 - p^2 \sin^2 \phi)}}$$  \hspace{1cm} (B.29)

Regarding the new limits, it will be seen from $\sin \psi = 2p^2 \sin^2 \phi - 1$, that if $\psi = 0$, $\sin \phi = 1/p\sqrt{2}$. Hence, the lower limit is given by

$$\phi_1 = \sin^{-1} \left( \frac{1}{p\sqrt{2}} \right)$$  \hspace{1cm} (B.30)

If $\psi = \psi_o$, $1 + \sin \psi = (1 + \sin \psi_o) \sin^2 \phi$, hence $\sin^2 \phi = 1$ and the upper limit is $\pi/2$. Therefore,

$$q = \int_{\phi_1}^{\pi/2} \frac{d\phi}{\sqrt{1 - p^2 \sin^2 \phi}}$$  \hspace{1cm} (B.31)

$$= K(p) - F(p, \phi_1)$$

In the above equation $K(p)$ and $F(p, \phi_1)$ are the complete and incomplete elliptic integrals of first kind respectively. The variable $p$ is the unknown quantity, which can be solved iteratively.

Tip deflection of the cantilever beam is obtained using the following relations

$$dx = ds \cos \psi$$  \hspace{1cm} (B.32)

$$dy = ds \sin \psi$$  \hspace{1cm} (B.33)
Integrating the above equations we will get the co-ordinates of the deformed profile

\[ dx = ds \cos \psi \]  \hspace{1cm} (B.34)

\[ x = \sqrt{\frac{EI}{F}} \int_{\phi}^{\phi_1} \frac{\cos \psi}{\sqrt{2(\sin \psi - \sin \psi_o)}} \, d\psi \]  \hspace{1cm} (B.35)

\[ = \sqrt{\frac{EI}{F}} \int_{\phi_1}^{\phi} 2p \sin \phi \, d\phi \]

\[ = 2p \sqrt{\frac{EI}{F}} (\cos \phi_1 - \cos \phi) \]

Now, consider the vertical displacement,

\[ dy = ds \sin \psi \]  \hspace{1cm} (B.36)

\[ y = \sqrt{\frac{EI}{F}} \int_{\phi_1}^{\phi} \frac{(2p^2 \sin^2 \phi - 1)}{\sqrt{(1 - p^2 \sin^2 \phi)}} \, d\phi \]  \hspace{1cm} (B.37)

The above equation can be arranged into the standard elliptic integral form in the following manner

\[ y = \sqrt{\frac{EI}{F}} \left( \int_{\phi_1}^{\phi} \frac{d\phi}{\sqrt{(1 - p^2 \sin^2 \phi)}} - 2 \int_{\phi_1}^{\phi} \sqrt{1 - p^2 \sin^2 \phi} \, d\phi \right) \]  \hspace{1cm} (B.38)

\[ = \sqrt{\frac{EI}{F}} \left[ F(p, \phi) - F(p, \phi_1) - 2E(p, \phi) + 2E(p, \phi_1) \right] \]

Tip deflection of the cantilever is obtained by substituting \( \phi = \pi/2 \) in the Eqs. (B.35) and (B.38).

\[ u_{tip} = L - 2p \sqrt{\frac{EI}{F}} \cos \phi_1 \]  \hspace{1cm} (B.39)

\[ w_{tip} = \sqrt{\frac{EI}{F}} \left[ K(p) - F(p, \phi_1) - 2E(p, \pi/2) + 2E(p, \phi_1) \right] \]  \hspace{1cm} (B.40)
Appendix C

Image Processing for Vision-Based Force-Sensing

For sensing the force with a vision-based force-sensor, it is required to compute the displacement field from the images captured before and after deformation under the unknown force. Displacement filed is computed if the correspondence between the deformed and undeformed configuration is established. This correspondence is established using any one of the following techniques.

1. Grid of markers

2. Speckle pattern

The *grid of markers* method establish the relation between the deformed and un-deformed configurations with the markings on the specimen. In MEMS devices this can be achieved by having control over the fabrication process. The speckle pattern is the natural characteristic of the surface which can be used as a reference to establish the relation. In this work, the *grid of markers* method is used. The details of this method are explained in the next section.
C.1 The grid of markers

In this method, to establish the relation between the material points in the deformed and undeformed configuration, a grid of markers are used. This grid of markers also serve as the nodes in the finite element mesh. The coordinates of the markers in the grid are obtained using the *Image-Pro Discovery* software. The method to obtain the nodal co-ordinates using the *Image-Pro Discovery* is summarized into the following steps.

1. Calibration
   
   (a) The *Image-Pro Discovery* software calibrated with the known dimension image captured under the given magnification. This is carried out using the *Spatial calibration wizard in the options menu*.

2. Extracting the nodal co-ordinates from the image
   
   (a) The image is loaded into the active window.

   (b) The reference calibration corresponding to the loaded image magnification is selected from the *spatial calibration* option which contains the pre-calibrated reference data of various magnifications.

   (c) Now point-wise data obtained from manual measurement menu with create point feature option as shown in Fig. C.1

   (d) The point data is exported to an excel file with *Input/output* option.
Figure C.1: Extracting the nodal data using the Image-Pro Discovery software
Appendix D

Finite Element Code

In this appendix, a two node co-rotational finite element code is presented for geometrically nonlinear analysis of frame structures. The code consist of the following primary M-files.

- **fem.m**: This is the main file required to be executed in the command window to solve the problem.
- **NR.m**: In this function, the finite element equilibrium equations are solved iteratively using Newton-Raphson method for every load increment.
- **gtanst.m**: The global tangent stiffness matrix and internal forces are computed with in this function.

In addition to the above mentioned M-files, it requires the following M-files and data files.

- The **matcut.m** and **vectcut.m** files to trim the specified degrees of freedom and the **update.m** file to update the displacement field during the incremental/iterative process.
- The **node.dat**, **element.dat**, **dispbc.dat** and **force.dat** files for supplying the necessary input data.

The format of the data files are also shown in this appendix.
Appendix D. Finite Element Code

D.1 fem.m

% Geometrically nonlinear finite element programme for beams
% Procedure: Total lagrangian approach
% References:
% 2. J.N. Reddy; "An introduction to Nonlinear finite element analysis"
clc
clear all
clf
global NX NY NNODE NELEM dispID ncon Axxx Dxxx dispVal fid1 NRT GTANS GFVS GFPS fid2 sna0 beta2 alphap
load force.dat
load node.dat
load element.dat
load dispbc.dat
NRT=0;
format long
% read nodal data
NNODE=size(node,1);
NX=node(:,2);
NY=node(:,3);
% read element data
NELEM=size(element,1);
ncon=element(:,[3 4]);
Axxx=element(:,5); Dxxx=element(:,6);
alphap = zeros(NELEM,1);
% Arrange force information into a force vector, F
F = zeros(3*NNODE,1); % Initialization
Nforce = size(force,1);
Fnode=force(1,2);
for i = 1:Nforce,
    dof = (force(i,2)-1)*3 + force(i,3);
    F(dof) = force(i,4);
end
fid1 = fopen('log_fem.dat','w');
fid2 = fopen('log_fem2.dat','w');
% fid2 = fopen('log_fem2.dat','w');
% dof1 = (force(1,2)-1)*3+1;
dof2= (force(1,2)-1)*3+2;

% Displacement boundary conditions
Nfix = size(dispbc,1);
j = 0;
for i = 1:Nfix,
    j = j + 1;
    dispID(j) = (dispbc(i,2)-1)*3+dispbc(i,3);
    dispVal(j) = dispbc(i,4);
end
[dispID sortIndex] = sort(dispID);
dispVal = dispVal(sortIndex);

beta2=zeros(NELEM,1);

% Define no of load steps
ld_steps=10;
F1=zeros(size(F,1),1);
NRit=200;
Finc=F/ld_steps;
U=zeros(3*NNODE,1);
flag=1;scale_factor = 1;
fc=0;
nn=0;UU=zeros(ld_steps,1);PP=zeros(ld_steps,1);
for i=1:ld_steps
    fprintf(fid1,'\nLOAD STEP NO : %g
',i);
    nn=nn+1;
    F1=F1+Finc;
    fprintf(fid1,'\n load step no: %i
',i)
    [U,flag]=NR(NRit,U,F1);
    if (flag == 0)
        fprintf(fid1,'\n Solution is not converged at load step %i \n
',i)
        break
    else
        fprintf(fid1,'\n
NODAL DISPLACEMENTS

');
        fprintf(fid1,'\t NODE NO.\tUx\t	Uy\t\ttheta 

');
        for j=1:NNODE
            fprintf(fid1,'\n %g\t%.16f\t%.16f\t %.16f\n',j,U(3*(j-1)+1),U(3*(j-1)+2),U(3*(j-1)+3));
        end
        U1 = scale_factor * U;
        Len=0;
        if (nn==2)
            for ip=1:NELEM
                fc= fc+1;
                pt1 = ncon(ip,1); pt2 = ncon(ip,2);
                dx1 = U1(3*(pt1-1)+1);
                dy1 = U1(3*(pt1-1)+2);
                dx2 = U1(3*(pt2-1)+1);
                dy2 = U1(3*(pt2-1)+2);
                Len= Len+(((NX(pt1)+dx1)-(NX(pt2)+dx2))^2+((NY(pt1)+dy1)-(NY(pt2)+dy2))^2)^(1/2);
Appendix D. Finite Element Code

D.2 NR.m

% function [U,flag]=NR(NRit,U,F);
global NX NY NNODE HELEM dispID ncon Axx Dxx dispVal fid1 NRT GTANS GFVS GFPS fid2 sna0 beta2 alphap F1=F;
ef=1e-6;
fprintf(fid1,'\t\tFORCE NORMS\t\t\n');
for i=1:NRit
    NRT=NRT+1;
    [GTS,GFV,F2] = gtanst(U,F1);
    delR=(F2-GFV);
    dudex=inv(GTS)*delR ;
    [U] =update(dudex,dispID,U,NNODE);
    if (NRT==1)
        for ie=1:HELEM
            sna0(ie)=-(U(3*(ncon(ie,2)-1)+1)-U(3*(ncon(ie,1)-1)+1))* (NY(ncon(ie,2))-NY(ncon(ie,1)))+(U(3*(ncon(ie,2)-1)+2)-U(3*(ncon(ie,1)-1)+2))* (NX(ncon(ie,2))-NX(ncon(ie,1)))+... 
    (U(3*(ncon(ie,2)-1)+1)*NY(ncon(ie,2))-U(3*(ncon(ie,1)-1)+1)*NY(ncon(ie,1)))-... 
    (U(3*(ncon(ie,2)-1)+1)*NX(ncon(ie,1))-U(3*(ncon(ie,2)-1)+1)*NX(ncon(ie,2)));... 
    end
    fprintf(fid1,'\n\nINTERNAL NODAL FORCES\n\n');
    fprintf(fid1,'\t NODE NO.\tFx\t\tFy\t\tM_z \n\n');
    for kk=1:NNODE
        fprintf(fid1,'
%10f\t%10f\t%10f\t %10f
',kk,GFVS(3*(kk-1)+1),GFVS(3*(kk-1)+2),GFVS(3*(kk-1)+3));
    end
end
fprintf(fid1,'\n\nTOTAL NO. OF NEWTON-RAPHSON ITERATIONS:%g\n',NRT);
fprintf(fid1,'\n*********** ANALYSIS HAS BEEN COMPLETED ***************\n');
fclose(fid1);fclose(fid2);
axis equal

hold on
plot([NX(pt1)+dx1 NX(pt2)+dx2], [NY(pt1)+dy1 NY(pt2)+dy2],'-k');
plot([NX(pt1)+dx1 NX(pt2)+dx2], [NY(pt1)+dy1 NY(pt2)+dy2],'.k');
plot([NX(pt1) NX(pt2)], [NY(pt1) NY(pt2)],'-k');
plot([NX(pt1) NX(pt2)], [NY(pt1) NY(pt2)],'.k');
end
end
if(nn==2)
nn=0;
end
xlabel('X');
ylabel('Y');
% check for convergence
if(i == 1)
    Ef0=(delR'*delR);
dudef0=dudef;
delR0=delR;
else
    if ((delR'*delR) <= ef* Ef0)
        flag=1;
        fprintf('
 current force norm %2.8g and intial force norm %2.8g',delR'*delR,Ef0)
        fprintf('
 Newton Raphson loop no : %i \n',i)
        fprintf(fid1,'NRIT:=%g \tCurrent Force Norm:=%g\n',i,delR'*delR);
        break
    end
end
fprintf('
 current force norm %2.8g and intial force norm %2.8g',delR'*delR,Ef0)
if(i == NRit)
    fprintf('
 solution not converged in %g number of iterations \n',NRit)
    flag=0;
    break
end
fprintf(fid1,'NRIT:=%g \tCurrent Force Norm:=%g\n',i,delR'*delR);

D.3 gtanst.m

function [GTS,GFV,F2] = gtanst(U,F1)
% this function forms the tangent stiffness matrix for the element and assembles it into global Matrix
global NX NY NNODE NELEM dispID ncon Axxx Dxxx dispVal fid1 NRT GTANS GFVS GFPS fid2 sma0 beta2 alphap
GTS = zeros(3*NNODE,3*NNODE);
GFV=zeros(3*NNODE,1);
GFPS=GFVS;
format short
for ie=1:NELEM
    eye1 = ncon(ie,1);
    jay = ncon(ie,2);
    x1=NX(eye1);x2=NX(jay);
    z1=NY(eye1);z2=NY(jay);
    u1=U(3*(eye1-1)+1);u2=U(3*(jay-1)+1);
    w1=U(3*(eye1-1)+2);w2=U(3*(jay-1)+2);
    theta1=U(3*(eye1-1)+3);theta2=U(3*(jay-1)+3);
    Axx=Axxx(ie);Dxx=Dxxx(ie);
    L0=sqrt((x2-x1)^2+(z2-z1)^2);
\begin{verbatim}
Ln=sqrt((x2+u2-x1-u1)^2+(z2+w2-z1-w1)^2);
c0=(x2-x1)/L0;
s0=(z2-z1)/L0;
c=(x2+u2-x1-u1)/Ln;s=(z2+w2-z1-w1)/Ln;
sna=c0*s-s0*c;ca=c0*c+s0*s;L=L0;
%find alpha based on the criteria given in thesis
if (NRT==1)
  alpha=0;
else
  if (sna0(ie)<0)
    alpha=atan2(sna,ca);
    if (alpha >0)
      alpha = -2*pi+alpha;
    end
    ad = abs(abs(alpha) - abs(alphap(ie)));
    if (ad > 2)
      alpha = atan2(sna,ca);
      sna0(ie) = -1*sna0(ie);
    end
  elseif (sna0(ie)>=0)
    alpha =atan2(sna,ca);
    if (alpha < 0)
      alpha = 2*pi + alpha;
    end
    ad = abs(abs(alpha) - abs(alphap(ie)));
    if (ad > 2)
      alpha = atan2(sna,ca);
      sna0(ie) = -1*sna0(ie);
    end
  end
end
alphap(ie) = alpha;

u_bar=Ln-L0;theta_b1=theta1-alpha;theta_b2=theta2-alpha;
B=[-c -s 0 c s 0
   -s/Ln c/Ln 1 s/Ln -c/Ln 0
   -s/Ln c/Ln 0 s/Ln -c/Ln 1];
z=[s -c 0 -s 0 0]';r=[-c -s 0 c s 0]';
N = Axx*(u_bar/L+1/15*theta_b1^2-1/30*theta_b1*theta_b2+1/15*theta_b2^2);
M1 = Axx*L*(u_bar/L+1/15*theta_b1^2-1/30*theta_b1*theta_b2+...1/15*theta_b2^2)*(2/15*theta_b1-1/30*theta_b2)+Dxx*(4*theta_b1+2*theta_b2)/L;
M2 = Axx*L*(u_bar/L+1/15*theta_b1^2-1/30*theta_b1*theta_b2+...1/15*theta_b2^2)*(2/15*theta_b2-1/30*theta_b1)+Dxx*(2*theta_b1+4*theta_b2)/L;
KL11 = Axx/L;
% KL12=0;
\end{verbatim}
\[
\begin{align*}
KL13 &= 0; \\
KL22 &= 4 \cdot Dxx/L; \\
KL23 &= 2 \cdot Dxx/L; \\
KL33 &= KL22; \\
KL12 &= Axx \cdot (2/15 \cdot \theta_b1 - 1/30 \cdot \theta_b2); \\
KL13 &= Axx \cdot (2/15 \cdot \theta_b2 - 1/30 \cdot \theta_b1); \\
KL22 &= Axx \cdot (2/15 \cdot \theta_b2 - 1/30 \cdot \theta_b1) \cdot 2^2/15 \cdot Axx \cdot L \cdot (u_bar/L + 1/15 \cdot \theta_b1^2 - 1/30 \cdot \theta_b1 \cdot \theta_b2 + \ldots \\
&\quad - 1/15 \cdot \theta_b2^2) + 2^2/15 \cdot Axx \cdot L \cdot (u_bar/L + \ldots \\
&\quad - 1/15 \cdot \theta_b1^2 - 1/30 \cdot \theta_b1 \cdot \theta_b2 + 1/15 \cdot \theta_b2^2) + 4 \cdot Dxx/L; \\
KL23 &= Axx \cdot (2/15 \cdot \theta_b2 - 1/30 \cdot \theta_b1) \cdot (2/15 \cdot \theta_b2 - 1/30 \cdot \theta_b1) \cdot 2^2/15 \cdot Axx \cdot L \cdot (u_bar/L + \ldots \\
&\quad - 1/15 \cdot \theta_b1^2 - 1/30 \cdot \theta_b1 \cdot \theta_b2 + 1/15 \cdot \theta_b2^2) + 2 \cdot Dxx/L; \\
KL33 &= Axx \cdot (2/15 \cdot \theta_b2 - 1/30 \cdot \theta_b1) \cdot (2/15 \cdot \theta_b2 - 1/30 \cdot \theta_b1) \cdot 2^2/15 \cdot Axx \cdot L \cdot (u_bar/L + \ldots \\
&\quad - 1/15 \cdot \theta_b1^2 - 1/30 \cdot \theta_b1 \cdot \theta_b2 + 1/15 \cdot \theta_b2^2) + 4 \cdot Dxx/L.
\end{align*}
\]

\[
KL12 = KL12; KL31 = KL13; KL32 = KL23;
\]

\[
KL = \begin{bmatrix} KL11 & KL12 & KL13 \\
KL21 & KL22 & KL23 \\
KL31 & KL32 & KL33 \end{bmatrix};
\]

\[
fint1 = [N; M1; M2];
\]

\[
fint = B' \cdot fint1;
\]

\[
Ktan1 = B' \cdot KL \cdot B;
\]

\[
Ktan2 = z' \cdot z / Ln; 
\]

\[
Ktan3 = (r' \cdot z + z' \cdot r') \cdot (M1 + M2) / Ln^2; 
\]

\[
Ktan = Ktan1 + Ktan2 + Ktan3;
\]

% Form ID matrix to assemble klocal into the global stiffness matrix, K.

\[
id1 = 3 \cdot (eye1-1) + 1; 
\]

\[
id2 = id1 + 1; 
\]

\[
id3 = id2 + 1; 
\]

\[
id4 = 3 \cdot (jny-1) + 1; 
\]

\[
id5 = id4 + 1; 
\]

\[
id6 = id5 + 1;
\]

% Form Global stiffness matrix

\[
GTS(id1, id1) = GTS(id1, id1) + Ktan(1, 1);
\]

\[
GTS(id1, id2) = GTS(id1, id2) + Ktan(1, 2);
\]

\[
GTS(id1, id3) = GTS(id1, id3) + Ktan(1, 3);
\]

\[
GTS(id1, id4) = GTS(id1, id4) + Ktan(1, 4);
\]

\[
GTS(id1, id5) = GTS(id1, id5) + Ktan(1, 5);
\]

\[
GTS(id1, id6) = GTS(id1, id6) + Ktan(1, 6);
\]

\[
GTS(id2, id1) = GTS(id2, id1) + Ktan(2, 1);
\]

\[
GTS(id2, id2) = GTS(id2, id2) + Ktan(2, 2);
\]

\[
GTS(id2, id3) = GTS(id2, id3) + Ktan(2, 3);
\]

\[
GTS(id2, id4) = GTS(id2, id4) + Ktan(2, 4);
\]

\[
GTS(id2, id5) = GTS(id2, id5) + Ktan(2, 5);
\]

\[
GTS(id2, id6) = GTS(id2, id6) + Ktan(2, 6);
\]
\texttt{GTS(id3,id1) = GTS(id3,id1) + K \tan(3,1);}
\texttt{GTS(id3,id2) = GTS(id3,id2) + K \tan(3,2);}
\texttt{GTS(id3,id3) = GTS(id3,id3) + K \tan(3,3);}
\texttt{GTS(id3,id4) = GTS(id3,id4) + K \tan(3,4);}
\texttt{GTS(id3,id5) = GTS(id3,id5) + K \tan(3,5);}
\texttt{GTS(id3,id6) = GTS(id3,id6) + K \tan(3,6);}
\texttt{GTS(id4,id1) = GTS(id4,id1) + K \tan(4,1);}
\texttt{GTS(id4,id2) = GTS(id4,id2) + K \tan(4,2);}
\texttt{GTS(id4,id3) = GTS(id4,id3) + K \tan(4,3);}
\texttt{GTS(id4,id4) = GTS(id4,id4) + K \tan(4,4);}
\texttt{GTS(id4,id5) = GTS(id4,id5) + K \tan(4,5);}
\texttt{GTS(id4,id6) = GTS(id4,id6) + K \tan(4,6);}
\texttt{GTS(id5,id1) = GTS(id5,id1) + K \tan(5,1);}
\texttt{GTS(id5,id2) = GTS(id5,id2) + K \tan(5,2);}
\texttt{GTS(id5,id3) = GTS(id5,id3) + K \tan(5,3);}
\texttt{GTS(id5,id4) = GTS(id5,id4) + K \tan(5,4);}
\texttt{GTS(id5,id5) = GTS(id5,id5) + K \tan(5,5);}
\texttt{GTS(id5,id6) = GTS(id5,id6) + K \tan(5,6);}
\texttt{GTS(id6,id1) = GTS(id6,id1) + K \tan(6,1);}
\texttt{GTS(id6,id2) = GTS(id6,id2) + K \tan(6,2);}
\texttt{GTS(id6,id3) = GTS(id6,id3) + K \tan(6,3);}
\texttt{GTS(id6,id4) = GTS(id6,id4) + K \tan(6,4);}
\texttt{GTS(id6,id5) = GTS(id6,id5) + K \tan(6,5);}
\texttt{GTS(id6,id6) = GTS(id6,id6) + K \tan(6,6);}
% Transform local force vector into global internal force vector
\texttt{GFV(id1,1) = GFV(id1,1) + fint(1);}
\texttt{GFV(id2,1) = GFV(id2,1) + fint(2);}
\texttt{GFV(id3,1) = GFV(id3,1) + fint(3);}
\texttt{GFV(id4,1) = GFV(id4,1) + fint(4);}
\texttt{GFV(id5,1) = GFV(id5,1) + fint(5);}
\texttt{GFV(id6,1) = GFV(id6,1) + fint(6);}
\texttt{end}
\texttt{GTANS=GTS;}
\texttt{GFVS=GFV;}
% Imposing displacement boundary conditions
% ------------------------------------------
% % dispID array contains the dof which are assigned specified values.
\texttt{[sm,sn] = size(dispID);}
\texttt{Ndbc = sn;}
\texttt{for nd=1:Ndbc}
\texttt{    for nr=1:3*NNODE-nd+1}
GFV(nr) = GFV(nr) - GTS(nr, dispID(nd) - nd + 1) * dispVal(nd);
end
GTS = matcut(GTS, dispID(nd) - nd + 1);
GFV = veccut(GFV, dispID(nd) - nd + 1);
F1 = veccut(F1, dispID(nd) - nd + 1);
end
F2 = F1;

D.4 update.m

% Function for updating the variables
function [U] = update(dudef, dispID, U, NNODE)
% Initialise the all the variable increments to zeros
[m, n] = size(dispID);
Ndbc = n;
du = zeros(3*NNODE, 1);
% For the specified displacement boundary conditions initialise du
for iu = 1:Ndbc,
    du(dispID(iu)) = 12345.12345;
end
% Assign incremental displacements to corresponding DOF
iuc = 0;
for iu = 1:3*NNODE,
    if du(iu) == 12345.12345
        iuc = iuc + 1;
    else
        du(iu) = dudef(iu - iuc);
    end
end
% Assign incremental displacement zero for specified DOF
for iu = 1:Ndbc,
    du(dispID(iu)) = 0.0;
end
% Increment the nodal displacements
U = U + du;

D.5 veccut.m

function D = veccut(C, i)
% To remove member i from C and return D with size 1 less.
[m, n] = size(C);
if m == 1
    d1 = C(1:i-1);
d2 = C(i+1:n);
D = [d1 d2];
end
if n == 1
    d1 = C(1:i-1);
    d2 = C(i+1:m);
    D = [d1; d2];
end

D.6 matcut.m

function D = mcut(C,i)
% To remove ith row and ith column from C (size: NxN) and
% return D (size: N-1xN-1)
[m,n] = size(C);
d1 = C(1:i-1,1:i-1);
d2 = C(1:i-1,i+1:n);
d3 = C(i+1:m,1:i-1);
d4 = C(i+1:m,i+1:n);
D = [d1 d2; d3 d4];

D.7 node.dat

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%NODE No. X-Coordinate Y-Coordinate %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
1 0.220116 8.39585
2 2.32694 8.23863
3 4.49665 8.14429
4 7.12233 8.06568
5 9.30776 8.00279
6 11.729 7.9399

D.8 element.dat

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Sr. Element First Second
% No. No. node node EA EI
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
1 1 1 2 20500 6.83333e-005
2 2 2 3 20500 6.83333e-005
3 3 3 4 20500 6.83333e-005
4 4 4 5 20500 6.83333e-005
5 5 5 6 20500 6.83333e-005
### D.9 dispbc.dat

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<th>No.</th>
<th>dof</th>
<th>dispVal</th>
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<td>1</td>
<td>1</td>
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<tr>
<td>2</td>
<td>1</td>
<td>2</td>
<td>0.00</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>3</td>
<td>0.00</td>
</tr>
</tbody>
</table>

### D.10 force.dat

<table>
<thead>
<tr>
<th>Sr. node</th>
<th>No.</th>
<th>dof</th>
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</thead>
<tbody>
<tr>
<td>1</td>
<td>5</td>
<td>2</td>
<td>10.00</td>
</tr>
</tbody>
</table>
References


[32] “Image-Pro Discovery”, Media Cybernetics, Inc. USA.